Eur. Phys. J. C 12, 137–160 (2000)
Digital Object Identifier (DOI) 10.1007/s100529900194

THE EUROPEAN
PHYSICAL JOURNAL C

A supersymmetric extension of the standard model with bilinear R-parity violation

Chao-hsi Chang[1,2], Tai-fu Feng[2]

[1] CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, P.R. China
[2] Institute of Theoretical Physics, Academia Sinica, P.O. Box 2735, Beijing 100080, P.R. China

Received: 26 April 1999 / Revised version: 28 June 1999 / Published online: 16 November 1999

Abstract. The minimum supersymmetric standard model with bilinear R-parity violation is studied systematically. As we consider low-energy supersymmetry, we examine the structure of the bilinear R-parity violating model carefully. We analyze the mixing of, e.g., Higgs bosons with sleptons, neutralinos with neutrinos and charginos with charged leptons in the model. Possible and some important physics results, e.g. that the lightest Higgs may be heavier than the weak Z-boson at tree level, are obtained. The Feynman rules for the model are derived in the 't Hooft–Feynman gauge, which is convenient if perturbative calculations are needed beyond the tree level.

1 Introduction

It is being increasingly realized by those engaged in supersymmetry (SUSY) research [1] that the principle of R-parity conservation, assumed to be sacrosanct in the prevalent research strategies, is not inviolable in practice. The R-parity of a particle is defined as $R=(-1)^{2S+3B+L}$ [2] and can be violated if either the baryon number (B) or lepton number (L) is not conserved. In recent years, extensive studies of supersymmetry which is characterized by bilinear R-parity violating terms in the superpotential and the nonzero vacuum expectation values (VEVs) of sneutrinos [3] have been undertaken. It stands as a simple supersymmetric (SUSY) model without R-parity which contains all particles of the standard model, and it can be arranged in such a way that there is no contradiction with the existing experimental data [4]. The impact of the R-parity violation on the low energy phenomenology is twofold in the model: (1) it affects the lepton number violation (LNV) explicitly; and (2) the bilinear R-parity violation terms in the superpotential and soft breaking terms generate nonzero vacuum expectation values for the sneutrino fields $\langle\tilde{\nu}_i\rangle\neq 0$ ($i=e,\mu,\tau$) and cause a new type of mixing, e.g. of neutrinos with neutralinos, of charged leptons with charginos, and of sleptons with Higgs.

The R-conserving superpotential for the minimal supersymmetric standard model (MSSM) has the following form in superfields:

$$\begin{aligned}\mathcal{W}_{\rm MSSM}=&\mu\varepsilon_{ij}\hat{H}_i^1\hat{H}_j^2\\&+l_I\varepsilon_{ij}\hat{H}_i^1\hat{L}_j^I\hat{R}^I-u_I(\hat{H}_1^2C^{JI*}\hat{Q}_2^J-\hat{H}_2^2\hat{Q}_1^I)\hat{U}^I\\&-d_I(\hat{H}_1^1\hat{Q}_2^I-\hat{H}_2^1C^{IJ}\hat{Q}_1^J)\hat{D}^I,\end{aligned}\tag{1}$$

where $\hat{H}^1$, $\hat{H}^2$ are Higgs superfields, $\hat{Q}^I$ and $\hat{L}^I$ are quark and lepton superfields respectively ($I=1,2,3$ is the index of the generation), and all of them are in a $SU(2)$ weak doublet. The rest are superfields; $\hat{U}^I$ and $\hat{D}^I$ for quarks and $\hat{R}^I$ for charged leptons are in a $SU(2)$ weak singlet. Here the indices i, j are contracted in a general way for the $SU(2)$ group, and C^{IJ} ($I,J=1,2,3$) are the elements of the CKM matrix. However, when R-breaking interactions are considered, the superpotential is modified as follows [5]:

$$\mathcal{W}=\mathcal{W}_{\rm MSSM}+\mathcal{W}_L+\mathcal{W}_B,\tag{2}$$

with

$$\begin{aligned}&\mathcal{W}_L=\varepsilon_{ij}[\lambda_{IJK}\hat{L}_i^I\hat{L}_j^J\hat{R}^K+\lambda'_{IJK}\hat{L}_i^I\hat{Q}_j^J\hat{D}^K+\epsilon_I\hat{H}_i^2\hat{L}_j^I],\\&\mathcal{W}_B=\lambda''_{IJK}\hat{U}^I\hat{D}^J\hat{D}^K.\end{aligned}\tag{3}$$

Since proton decay experiments determine a very stringent limit on the baryon number violation [25], we suppress the term $\mathcal{W}_B$ completely. The first two terms in $\mathcal{W}_L$ have received a lot of attention recently, and restrictions have been derived on them from existing experimental data [6]. However, the term $\epsilon_I\varepsilon_{ij}\hat{H}_i^2\hat{L}_j^J$ is also a viable agent for R-parity breaking. It is particularly interesting because it can result in observable effects that are not seen with the trilinear terms alone. One of these distinctive effects is that the lightest neutralino can decay invisibly into three neutrinos at the tree level, which is not possible if only the trilinear terms in $\mathcal{W}_L$ are presented. The significance of such a bilinear R-parity violating interaction is further emphasized by the following observations:

– Although it may seem possible to rotate the $\hat{H}_2\hat{L}$ terms away by redefining the lepton and Higgs superfields [7], their effect is bound to show up via the soft breaking terms.

- Even if one may rotate these terms away at one energy scale, they will reappear at another one as the couplings evolve radiatively [8].
- The bilinear terms give rise to the trilinear terms at the one-loop level [9].
- It has been argued that if one wants to subsume R-parity violation in a grand unified theory (GUT), then the trilinear R-parity violating terms become rather small in magnitude (about $\sim 10^{-3}$) [10]. However, the superrenormalizable bilinear terms are not subjected to such requirements.

In this paper we will have $\epsilon_I \varepsilon_{ij} \hat{L}^I_i H^2_j$ as the only R-parity violating terms to study the phenomenology of the model. The paper is organized is follows. In Sect. 2 we will describe the basic ingredients of supersymmetry with bilinear R-parity violation. The mass matrices of the CP-even, CP-odd and charged Higgs are derived. Some interesting relations for CP-even and CP-odd Higgs masses are obtained. For completeness we also give the mixing matrices of charginos with charged leptons and of neutralinos with neutrinos. In Sect. 3 we will give the Feynman rules for the interaction of the Higgs bosons (sleptons) with the gauge bosons, and of the charginos and neutralinos with gauge bosons or Higgs bosons (sleptons). The self-interactions of the Higgs and the chargino (neutralino)–squark–quark interactions are also given. In Sect. 4 we will analyze the particle spectrum by a numerical method under a few assumptions about the parameters in the model. We find that the possibility with large values for ϵ_3 and $\upsilon_{\tilde{\nu}_\tau}$ still survives under strong experimental restrictions for the masses of the τ-neutrino ($m_{\nu_\tau} \leq 20\,\mathrm{MeV}$) and of the τ-lepton ($m_\tau = 1.77\,\mathrm{GeV}$). We will close our discussions with comments on the model.

2 SUSY with bilinear R-parity violation

As stated above, we are to consider a superpotential of the form

$$\begin{aligned}\mathcal{W} = &\mu\varepsilon_{ij}\hat{H}^1_i\hat{H}^2_j + l_I\varepsilon_{ij}\hat{H}^1_i\hat{L}^I_j\hat{R}^I \\ &-u^I(\hat{H}^2_1 C^{JI*}\hat{Q}^J_2 - \hat{H}^2_2\hat{Q}^I_1)\hat{U}^I \\ &-d^I(\hat{H}^1_1\hat{Q}^I_2 - \hat{H}^1_2 C^{IJ}\hat{Q}^J_1)\hat{D}^I + \epsilon_I\varepsilon_{ij}\hat{H}^2_i\hat{L}^I_j, \quad (4)\end{aligned}$$

where μ, ϵ_I are the parameters with units of mass, u^I, d^I and l^I are the Yukawa couplings as in the MSSM with R-parity. In order to break the supersymmetry, we introduce the soft SUSY-breaking terms

$$\begin{aligned}\mathcal{L}_{\mathrm{soft}} = &-m^2_{H^1}H^{1*}_i H^1_i - m^2_{H^2}H^{2*}_i H^2_i - m^2_{L^I}\tilde{L}^{I*}_i\tilde{L}^I_i \\ &-m^2_{R^I}\tilde{R}^{I*}\tilde{R}^I - m^2_{Q^I}\tilde{Q}^{I*}_i\tilde{Q}^I_i - m^2_{D^I}\tilde{D}^{I*}\tilde{D}^I \\ &-m^2_{U^I}\tilde{U}^{I*}\tilde{U}^I + (m_1\lambda_B\lambda_B + m_2\lambda^i_A\lambda^i_A \\ &+m_3\lambda^a_G\lambda^a_G + \mathrm{h.c.}) + \{B\mu\varepsilon_{ij}H^1_i H^2_j \\ &+B_I\epsilon_I\varepsilon_{ij}H^2_i\tilde{L}^I_j + \varepsilon_{ij}l_{sI}\mu H^1_i\tilde{L}^I_j\tilde{R}^I \\ &+d_{sI}\mu(-H^1_1\tilde{Q}^I_2 + C^{IK}H^1_2\tilde{Q}^K_1)\tilde{D}^I \\ &+u_{sI}\mu(-C^{KI*}H^2_1\tilde{Q}^I_2 + H^2_2\tilde{Q}^I_1)\tilde{U}^I + \mathrm{h.c.}\}, \quad (5)\end{aligned}$$

where $m^2_{H^1}, m^2_{H^2}, m^2_{L^I}, m^2_{R^I}, m^2_{Q^I}, m^2_{D^I}$ and $m^2_{U^I}$ are parameters with units of mass squared, while m_3, m_2 and m_1 denote the masses of λ^a_G, λ^i_A and λ_B, the $SU(3) \times SU(2) \times U(1)$ gauginos. B and B_I are free parameters with units of mass. d_{sI}, u_{sI}, l_{sI} ($I = 1, 2, 3$) are the soft breaking parameters that give the mass splitting between the quarks, leptons and their supersymmetric partners. The other parts in the model (like the gauge, matter, and the gauge–matter interactions) are the same as in the MSSM with R-parity. We will not repeat them here.

Thus the scalar potential of the model can be written as

$$\begin{aligned}V &= \sum_i \left|\frac{\partial\mathcal{W}}{\partial A_i}\right|^2 + V_D + V_{\mathrm{soft}} \\ &= V_F + V_D + V_{\mathrm{soft}}, \quad (6)\end{aligned}$$

where A_i denote the scalar fields, V_D are the usual D-terms, and V_{soft} are the SUSY soft breaking terms given in (5). When we use the superpotential (4) and the soft breaking terms (5), we can write down the scalar potential precisely.

The electroweak symmetry is broken spontaneously when the two Higgs doublets H^1, H^2 and the sleptons acquire nonzero vacuum expectation values (VEVs):

$$H^1 = \begin{pmatrix}\frac{1}{\sqrt{2}}(\chi^0_1 + \upsilon_1 + \mathrm{i}\phi^0_1) \\ H^1_2\end{pmatrix}, \quad (7)$$

$$H^2 = \begin{pmatrix}H^2_1 \\ \frac{1}{\sqrt{2}}(\chi^0_2 + \upsilon_2 + \mathrm{i}\phi^0_2)\end{pmatrix}, \quad (8)$$

and

$$\tilde{L}^I = \begin{pmatrix}\frac{1}{\sqrt{2}}(\chi^0_{\tilde{\nu}_I} + \upsilon_{\tilde{\nu}_I} + \mathrm{i}\phi^0_{\tilde{\nu}_I}) \\ \tilde{L}^I_2\end{pmatrix}, \quad (9)$$

where $\tilde{L}^I$ denote the slepton doublets and $I = e, \mu, \tau$ the generation indices of the leptons. From (5) and (6) we find that the scalar potential includes the linear terms as follows:

$$V_{\mathrm{tadpole}} = t^0_1\chi^0_1 + t^0_2\chi^0_2 + t^0_{\tilde{\nu}_e}\chi^0_{\tilde{\nu}_e} + t^0_{\tilde{\nu}_\mu}\chi^0_{\tilde{\nu}_\mu} + t^0_{\tilde{\nu}_\tau}\chi^0_{\tilde{\nu}_\tau}, \quad (10)$$

where

$$\begin{aligned}t^0_1 = &\frac{1}{8}(g^2 + g'^2)\upsilon_1\left(\upsilon^2_1 - \upsilon^2_2 + \sum_I \upsilon^2_{\tilde{\nu}_I}\right) + |\mu|^2\upsilon_1 \\ &+m^2_{H^1}\upsilon_1 - B\mu\upsilon_2 - \sum_I \mu\epsilon_I\upsilon_I, \\ t^0_2 = &-\frac{1}{8}(g^2 + g'^2)\upsilon_2\left(\upsilon^2_1 - \upsilon^2_2 + \sum_I \upsilon^2_{\tilde{\nu}_I}\right) + |\mu|^2\upsilon_2 \\ &+m^2_{H^2}\upsilon_2 - B\mu\upsilon_1 + \sum_I \epsilon^2_I\upsilon_2 + \sum_I B_I\epsilon_I\upsilon_I, \\ t^0_{\tilde{\nu}_I} = &\frac{1}{8}(g^2 + g'^2)\upsilon_{\tilde{\nu}_I}\left(\upsilon^2_1 - \upsilon^2_2 + \sum_I \upsilon^2_{\tilde{\nu}_I}\right) + m^2_{L^I}\upsilon_{\tilde{\nu}_I} \\ &+\epsilon_I\sum_J \epsilon_J\upsilon_{\tilde{\nu}_J} - \mu\epsilon_I\upsilon_1 + B_I\epsilon_I\upsilon_2. \quad (11)\end{aligned}$$

Here t_i^0 $(i = 1, 2, \tilde{\nu}_e, \tilde{\nu}_\mu, \tilde{\nu}_\tau)$ are tadpoles at the tree level, and the VEVs of the neutral scalar fields should satisfy the conditions $t_i^0 = 0$ $(i = 1, 2, \tilde{\nu}_e, \tilde{\nu}_\mu, \tilde{\nu}_\tau)$. Therefore one can obtain

$$\begin{aligned}
m_{H^1}^2 &= -\left(|\mu|^2 - \sum_I \epsilon_I \mu \frac{\upsilon_{\tilde{\nu}_I}}{\upsilon_1} - B\mu \frac{\upsilon_2}{\upsilon_1}\right. \\
&\quad \left. + \frac{1}{8}(g^2 + g'^2)\left(\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon_{\tilde{\nu}_I}^2\right)\right), \\
m_{H^2}^2 &= -\left(|\mu|^2 + \sum_I \epsilon_I^2 + \sum_I B_I \epsilon_I \frac{\upsilon_{\tilde{\nu}_I}}{\upsilon_2} - B\mu \frac{\upsilon_1}{\upsilon_2}\right. \\
&\quad \left. - \frac{1}{8}(g^2 + g'^2)\left(\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon_{\tilde{\nu}_I}^2\right)\right), \\
m_{L^I}^2 &= -\left(\frac{1}{8}(g^2 + g'^2)\left(\upsilon_1^2 - \upsilon_2^2 - \sum_I \upsilon_{\tilde{\nu}_I}^2\right)\right. \\
&\quad \left. + \epsilon_I \sum_J \epsilon_J \frac{\upsilon_{\tilde{\nu}_J}}{\upsilon_{\tilde{\nu}_I}} - \epsilon_I \mu \frac{\upsilon_1}{\upsilon_{\tilde{\nu}_I}} + B_I \epsilon_I \frac{\upsilon_2}{\upsilon_{\tilde{\nu}_I}}\right) \\
&\quad (I = e, \mu, \tau).
\end{aligned} \tag{12}$$

For convenience we will call all these scalar bosons (H^1, H^2 and $\tilde{L}^I$) Higgs below. Now we will give the Higgs boson mass matrix explicitly. For the scalar sector, the mass squared matrices may be obtained by

$$\mathcal{M}_{ij}^2 = \left.\frac{\partial^2 V}{\partial \phi_i \phi_j}\right|_{\text{minimum}}. \tag{13}$$

Here "minimum" means that the values are evaluated at $\langle H_1^1 \rangle = \upsilon_1/\sqrt{2}$, $\langle H_2^2 \rangle = \upsilon_2/\sqrt{2}$, $\langle \tilde{L}_1^I \rangle = \upsilon_{\tilde{\nu}_I}/\sqrt{2}$ and $\langle A_i \rangle = 0$ (A_i represent all other scalar fields). Thus the squared mass matrices of both the CP-even and the CP-odd scalar bosons are 5×5, whereas the matrix of the charged Higgs is 8×8.

2.1 The squared mass matrices of Higgs

From the scalar potential (6) we can find the mass terms

$$\mathcal{L}_{\mathrm{m}}^{\mathrm{even}} = -\Phi_{\mathrm{even}}^{\dagger} \mathcal{M}_{\mathrm{even}}^2 \Phi_{\mathrm{even}}, \tag{14}$$

where "current" CP-even Higgs fields $\Phi_{\mathrm{even}}^{\mathrm{T}} = (\chi_1^0, \chi_2^0, \chi_{\tilde{\nu}_e}^0, \chi_{\tilde{\nu}_\mu}^0, \chi_{\tilde{\nu}_\tau}^0)$. The mass matrix in (14) is

$$\mathcal{M}_{\mathrm{even}}^2 = \begin{pmatrix}
r_{11} & -e_{12} - B\mu & e_{13} - \mu\epsilon_1 & e_{14} - \mu\epsilon_2 & e_{15} - \mu\epsilon_3 \\
-e_{12} - B\mu & r_{22} & -e_{23} + B_1\epsilon_1 & -e_{24} + B_2\epsilon_2 & -e_{25} + B_3\epsilon_3 \\
e_{13} - \mu\epsilon_1 & -e_{23} + B_1\epsilon_1 & r_{33} & e_{34} + \epsilon_1\epsilon_2 & e_{35} + \epsilon_1\epsilon_3 \\
e_{14} - \mu\epsilon_2 & -e_{24} + B_2\epsilon_2 & e_{34} + \epsilon_1\epsilon_2 & r_{44} & e_{45} + \epsilon_2\epsilon_3 \\
e_{15} - \mu\epsilon_3 & -e_{25} + B_3\epsilon_3 & e_{35} + \epsilon_1\epsilon_3 & e_{45} + \epsilon_2\epsilon_3 & r_{55}
\end{pmatrix}, \tag{15}$$

with notations

$$\begin{aligned}
r_{11} &= \frac{g^2 + g'^2}{8}\left(3\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon_{\tilde{\nu}_I}^2\right) + |\mu|^2 + m_{H^1}^2 \\
&= \frac{g^2 + g'^2}{4}\upsilon_1^2 + \sum_I \mu\epsilon_I \frac{\upsilon_{\tilde{\nu}_I}}{\upsilon_1} + B\mu \frac{\upsilon_2}{\upsilon_1}, \\
r_{22} &= \frac{g^2 + g'^2}{8}\left(-\upsilon_1^2 + 3\upsilon_2^2 - \sum_I \upsilon_{\tilde{\nu}_I}^2\right) + |\mu|^2 \\
&\quad + \sum_I \epsilon_I^2 + m_{H^2}^2 \\
&= \frac{g^2 + g'^2}{4}\upsilon_2^2 + B\mu \frac{\upsilon_1}{\upsilon_2} - \sum_I B_I \epsilon_I \frac{\upsilon_{\tilde{\nu}_I}}{\upsilon_2}, \\
r_{33} &= \frac{g^2 + g'^2}{8}\left(\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon_{\tilde{\nu}_I}^2 + 2\upsilon_{\tilde{\nu}_e}^2\right) + \epsilon_1^2 + m_{L^1}^2 \\
&= \frac{g^2 + g'^2}{4}\upsilon_{\tilde{\nu}_e}^2 + \mu\epsilon_1 \frac{\upsilon_1}{\upsilon_{\tilde{\nu}_e}} - B_1\epsilon_1 \frac{\upsilon_2}{\upsilon_{\tilde{\nu}_e}} \\
&\quad - \epsilon_1\epsilon_2 \frac{\upsilon_{\tilde{\nu}_\mu}}{\upsilon_{\tilde{\nu}_e}} - \epsilon_1\epsilon_3 \frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_{\tilde{\nu}_e}}, \\
r_{44} &= \frac{g^2 + g'^2}{8}\left(\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon_{\tilde{\nu}_I}^2 + 2\upsilon_{\tilde{\nu}_\mu}^2\right) + \epsilon_2^2 + m_{L^2}^2 \\
&= \frac{g^2 + g'^2}{4}\upsilon_{\tilde{\nu}_\mu}^2 + \mu\epsilon_2 \frac{\upsilon_1}{\upsilon_{\tilde{\nu}_\mu}} - B_2\epsilon_2 \frac{\upsilon_2}{\upsilon_{\tilde{\nu}_\mu}} \\
&\quad - \epsilon_1\epsilon_2 \frac{\upsilon_{\tilde{\nu}_e}}{\upsilon_{\tilde{\nu}_\mu}} - \epsilon_2\epsilon_3 \frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_{\tilde{\nu}_\mu}}, \\
r_{55} &= \frac{g^2 + g'^2}{8}\left(\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon_{\tilde{\nu}_I}^2 + 2\upsilon_{\tilde{\nu}_\tau}^2\right) + \epsilon_3^2 + m_{L^3}^2 \\
&= \frac{g^2 + g'^2}{4}\upsilon_{\tilde{\nu}_\tau}^2 + \mu\epsilon_3 \frac{\upsilon_1}{\upsilon_{\tilde{\nu}_\tau}} - B_3\epsilon_3 \frac{\upsilon_2}{\upsilon_{\tilde{\nu}_\tau}} \\
&\quad - \epsilon_1\epsilon_3 \frac{\upsilon_{\tilde{\nu}_e}}{\upsilon_{\tilde{\nu}_\tau}} - \epsilon_2\epsilon_3 \frac{\upsilon_{\tilde{\nu}_\mu}}{\upsilon_{\tilde{\nu}_\tau}},
\end{aligned} \tag{16}$$

and

$$\begin{aligned}
e_{12} &= \frac{g^2 + g'^2}{4}\upsilon_1\upsilon_2, & e_{13} &= \frac{g^2 + g'^2}{4}\upsilon_1\upsilon_{\tilde{\nu}_e}, \\
e_{14} &= \frac{g^2 + g'^2}{4}\upsilon_1\upsilon_{\tilde{\nu}_\mu}, & e_{15} &= \frac{g^2 + g'^2}{4}\upsilon_1\upsilon_{\tilde{\nu}_\tau}, \\
e_{23} &= \frac{g^2 + g'^2}{4}\upsilon_2\upsilon_{\tilde{\nu}_e}, & e_{24} &= \frac{g^2 + g'^2}{4}\upsilon_2\upsilon_{\tilde{\nu}_\mu}, \\
e_{25} &= \frac{g^2 + g'^2}{4}\upsilon_2\upsilon_{\tilde{\nu}_\tau}, & e_{34} &= \frac{g^2 + g'^2}{4}\upsilon_{\tilde{\nu}_e}\upsilon_{\tilde{\nu}_\mu}, \\
e_{35} &= \frac{g^2 + g'^2}{4}\upsilon_{\tilde{\nu}_e}\upsilon_{\tilde{\nu}_\tau}, & e_{45} &= \frac{g^2 + g'^2}{4}\upsilon_{\tilde{\nu}_\mu}\upsilon_{\tilde{\nu}_\tau}.
\end{aligned} \tag{17}$$

Here in order to obtain the above mass matrix, (12) is used. The physical CP-even Higgs can be obtained by

$$H_i^0 = \sum_{j=1}^{5} Z_{\mathrm{even}}^{ij} \chi_j^0, \tag{18}$$

where $Z_{i,j}^{\mathrm{even}}$ $(i, j = 1, 2, 3, 4, 5)$ are the elements of the matrix that converts mass matrix (16) into a diagonal matrix, i.e. it translates the current fields into physical fields (corresponding to the eigenstates of the mass matrix).

In the current basis $\Phi^{\mathrm{T}}_{\mathrm{odd}} = (\phi_1^0, \phi_2^0, \phi_{\tilde{\nu}_e}^0, \phi_{\tilde{\nu}_\mu}^0, \phi_{\tilde{\nu}_\tau}^0)$, the mass matrix for the CP-odd scalar fields can be written as

$$\mathcal{M}^2_{\mathrm{odd}} = \begin{pmatrix} s_{11} & B\mu & -\mu\epsilon_1 & -\mu\epsilon_2 & -\mu\epsilon_3 \\ B\mu & s_{22} & -B_1\epsilon_1 & -B_2\epsilon_2 & -B_3\epsilon_3 \\ -\mu\epsilon_1 & -B_1\epsilon_1 & s_{33} & \epsilon_1\epsilon_2 & \epsilon_1\epsilon_3 \\ -\mu\epsilon_2 & -B_2\epsilon_2 & \epsilon_1\epsilon_2 & s_{44} & \epsilon_2\epsilon_3 \\ -\mu\epsilon_3 & -B_3\epsilon_3 & \epsilon_1\epsilon_3 & \epsilon_2\epsilon_3 & s_{55} \end{pmatrix}, \quad (19)$$

with

$$\begin{aligned} s_{11} &= \frac{g^2+g'^2}{8}(\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon_{\tilde{\nu}_I}^2) + \mu^2 + m^2_{H^1} \\ &= \sum_I \mu\epsilon_I \frac{\upsilon_{\tilde{\nu}_I}}{\upsilon_1} + B\mu\frac{\upsilon_2}{\upsilon_1}, \\ s_{22} &= -\frac{g^2+g'^2}{8}(\upsilon_1^2 - \upsilon_2^2 + \upsilon_{\tilde{\nu}_I}^2) + \mu^2 + \sum_I \epsilon_I^2 + m^2_{H^2} \\ &= B\mu\frac{\upsilon_1}{\upsilon_2} - \sum_I B_I\epsilon_I\frac{\upsilon_{\tilde{\nu}_I}}{\upsilon_2}, \\ s_{33} &= \frac{g^2+g'^2}{8}(\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon_{\tilde{\nu}_I}^2) + \epsilon_1^2 + m^2_{L^1} \\ &= \mu\epsilon_1\frac{\upsilon_1}{\upsilon_{\tilde{\nu}_e}} - B_1\epsilon_1\frac{\upsilon_2}{\upsilon_{\tilde{\nu}_e}} - \epsilon_1\epsilon_2\frac{\upsilon_{\tilde{\nu}_\mu}}{\upsilon_{\tilde{\nu}_e}} - \epsilon_1\epsilon_3\frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_{\tilde{\nu}_e}}, \\ s_{44} &= \frac{g^2+g'^2}{8}(\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon_{\tilde{\nu}_I}^2) + \epsilon_2^2 + m^2_{L^2} \\ &= \mu\epsilon_2\frac{\upsilon_1}{\upsilon_{\tilde{\nu}_\mu}} - B_2\epsilon_2\frac{\upsilon_2}{\upsilon_{\tilde{\nu}_\mu}} - \epsilon_1\epsilon_2\frac{\upsilon_{\tilde{\nu}_e}}{\upsilon_{\tilde{\nu}_\mu}} - \epsilon_2\epsilon_3\frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_{\tilde{\nu}_\mu}}, \\ s_{55} &= \frac{g^2+g'^2}{8}(\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon_{\tilde{\nu}_I}^2) + \epsilon_3^2 + m^2_{L^3} \\ &= \mu\epsilon_3\frac{\upsilon_1}{\upsilon_{\tilde{\nu}_\tau}} - B_3\epsilon_3\frac{\upsilon_2}{\upsilon_{\tilde{\nu}_\tau}} - \epsilon_1\epsilon_3\frac{\upsilon_{\tilde{\nu}_e}}{\upsilon_{\tilde{\nu}_\tau}} - \epsilon_2\epsilon_3\frac{\upsilon_{\tilde{\nu}_\mu}}{\upsilon_{\tilde{\nu}_\tau}}. \end{aligned} \quad (20)$$

From (19) and (20) one can find that the neutral Goldstone boson (with zero mass) can be given as [11]

$$\begin{aligned} G^0 \equiv H_6^0 &= \sum_{i=1}^5 Z^{1,i}_{\mathrm{odd}}\phi_i^0 \\ &= \frac{1}{\upsilon}(\upsilon_1\phi_1^0 - \upsilon_2\phi_2^0 + \upsilon_{\tilde{\nu}_e}\phi^0_{\tilde{\nu}_e} + \upsilon_{\tilde{\nu}_\mu}\phi^0_{\tilde{\nu}_\mu} + \upsilon_{\tilde{\nu}_\tau}\phi^0_{\tilde{\nu}_\tau}), \end{aligned} \quad (21)$$

which is indispensable for spontaneous EW gauge symmetry breaking. Here $\upsilon = (\upsilon_1^2 + \upsilon_2^2 + \sum_I \upsilon_{\tilde{\nu}_I}^2)^{1/2}$, and the mass of the Z-boson $M_Z = ((g^2+g'^2)^{1/2}/2)\upsilon$ is the same as in R-parity conserved MSSM. The other four physical neutral bosons can be written as

$$H^0_{5+i}(i=2,3,4,5) = \sum_{j=1}^5 Z^{i,j}_{\mathrm{odd}}\phi_j^0, \quad (22)$$

where $Z^{\mathrm{odd}}_{i,j}$ $(i,j = 1,2,3,4,5)$ is the matrix that converts the current fields into the physical eigenstates.

From the eigenvalue equations one can find two independent relations:

$$\begin{aligned} &\sum_{i=1}^5 m^2_{H_i^0} = \sum_{i=2}^5 m^2_{H^0_{5+i}} + m_Z^2, \\ &\prod_{i=1}^5 m^2_{H_i^0} = \left[\frac{\upsilon_1^2 - \upsilon_2^2 + \sum_{I=1}^3 \upsilon^2_{\tilde{\nu}_I}}{\upsilon^2}\right]^2 m_Z^2 \prod_{i=2}^5 m^2_{H^0_{5+i}}. \end{aligned} \quad (23)$$

If we introduce the following notations:

$$\begin{aligned} \upsilon_1 &= \upsilon\cos\beta\cos\theta_\upsilon, \\ \upsilon_2 &= \upsilon\sin\beta, \\ \sqrt{\sum_{I=1}^3 \upsilon^2_{\nu_I}} &= \upsilon\cos\beta\sin\theta_\upsilon, \end{aligned} \quad (24)$$

the second relation of (23) can be written as

$$\prod_{i=1}^5 m^2_{H_i^0} = \cos^2 2\beta m_Z^2 \prod_{i=2}^5 m^2_{H^0_{5+i}}. \quad (25)$$

The first equation of (23) is also obtained in [12]. Whereas we consider that the second equation of (23) is also important, the two equations are independent restrictions on the masses of neutral Higgs bosons. For instance, from (23) and (25) we have an upper limit on the mass of the lightest Higgs at tree level in the model:

$$m^2_{H_1^0} \le m^2_{H_n^0}\left(\frac{m_Z^2\cos^2 2\beta}{m^2_{H_n^0}}\right)^{\frac{1}{n-1}} \frac{1 - \frac{1}{n-1}\frac{m_Z^2}{m^2_{H_n^0}}}{1 - \frac{1}{n-1}\left(\frac{m_Z^2\cos^2 2\beta}{m^2_{H_n^0}}\right)^{\frac{1}{n-1}}}, \quad (26)$$

where $n \ge 2$ is the number of the CP-even Higgs, $m_{H_1^0}$ is the mass of the lightest one among them, and $m_{H_n^0}$ is the mass of the heaviest one. Some remarks should be made about (26):

- From (23) and (25) we can find $m^2_{H_n^0} \ge m_Z^2$.
- When $n = 2$ or $m^2_{H_1^0} = \cdots = m^2_{H_n^0} = m^2_{H^0_{n+2}} = \cdots = m^2_{H^0_{n+n}} = m_Z^2$, $\cos^2 2\beta = 1$, "=" is established.
- In the case of MSSM with R-parity ($n = 2$),

$$m_{H_1^0} = m_Z^2\cos^2 2\beta \frac{1 - \frac{m_Z^2}{m^2_{H_2^0}}}{1 - \frac{m_Z^2}{m^2_{H_2^0}}\cos^2 2\beta} \le m_Z^2\cos^2 2\beta$$

  is recovered.

So when $n > 2$, we cannot impose an upper limit on $m_{H_1^0}$ as we can for the R-parity conserved MSSM at the tree level, namely, for the later it is just the $n = 2$ case [19]:

$$m^2_{H_1^0} \le m_Z^2\cos^2 2\beta \le m_Z^2. \quad (27)$$

When one considers experimental data, one cannot rule out large ϵ_I ($I = 1, 2, 3$) [21,29]. Furthermore, even if $\epsilon_I \ll \mu$, we still have no reason to assume that $B_I \epsilon_I \ll B\mu$ in general. In the MSSM with R-parity, the radiative corrections to the mass of the lightest Higgs are large [26] when complete one-loop corrections and leading two-loop corrections of $\mathcal{O}(\alpha\alpha_s)$ are included; the limit on the lightest Higgs mass, $m_{H_1^0} \leq 132\,\mathrm{GeV}$, is given in [27]. In the MSSM without R-parity, there is not such a stringent restriction on the lightest Higgs even at the tree level.

When we take the "current" basis $\Phi_c = (H_2^{1*}, H_1^2, \tilde{L}_2^{1*}, \tilde{L}_2^{2*}, \tilde{L}_2^{3*}, \tilde{R}^1, \tilde{R}^2, \tilde{R}^3)$ and (6), we can find the following mass terms in the Lagrangian:

$$\mathcal{L}_\mathrm{m}^\mathrm{C} = -\Phi_\mathrm{c}^\dagger \mathcal{M}_\mathrm{c}^2 \Phi_\mathrm{c}. \tag{28}$$

$\mathcal{M}_\mathrm{c}^2$ is given in Appendix A. When we diagonalize the mass matrix for the charged Higgs bosons, we obtain the zero mass Goldstone boson state:

$$H_1^+ = \sum_{i=1}^{8} Z_\mathrm{c}^{1,i} \Phi_\mathrm{c}^i = \frac{1}{\upsilon}(\upsilon_1 H_2^{1*} - \upsilon_2 H_1^2 + \upsilon_{\tilde{\nu}_e} \tilde{L}_2^{1*} + \upsilon_{\tilde{\nu}_\mu} \tilde{L}_2^{2*} + \upsilon_{\tilde{\nu}_\tau} \tilde{L}_2^{3*}) \tag{29}$$

together with the charge conjugate state H_1^-, which are indispensable to break electroweak symmetry and which give the masses of the $W^\pm$ bosons. With the transformation matrix Z_c^{ij} (which converts the current fields into the physical eigenstates basis), the other seven physical eigenstates H_i^+ ($i = 2, 3, 4, 5, 6, 7, 8$) can be expressed as

$$H_i^+ = \sum_{j=1}^{8} Z_\mathrm{c}^{i,j} \Phi_j^\mathrm{c} \qquad (i, j = 1, \cdots, 8). \tag{30}$$

2.2 The mixing of neutralinos with neutrinos

Due to the lepton number violation in the MSSM without R-parity, new and interesting mixing of neutralinos with neutrinos and of charginos with charged leptons may happen. The part of the Lagrangian which is responsible for the mixing of neutralinos with neutrinos is

$$\mathcal{L}_{\kappa_i^0}^\mathrm{mass} = \left\{ \mathrm{i}g \frac{1}{\sqrt{2}} \tau_{ij}^i \lambda_A^i \psi_j A_i^* + \mathrm{i}g' \sqrt{2} Y_i \lambda_B \psi_i A_i^* - \frac{1}{2} \frac{\partial^2 \mathcal{W}}{\partial A_i \partial A_j} \psi_i \psi_j + \mathrm{h.c.} \right\} + m_1(\lambda_B \lambda_B + \mathrm{h.c.}) + m_2(\lambda_A^i \lambda_A^i + \mathrm{h.c.}), \tag{31}$$

where $\mathcal{W}$ is given by (4). $\tau^i/2$ and Y_i are the generators of the $SU(2) \times U(1)$ gauge group, and ψ_i, A_i stand for generic two-component fermions and scalar fields. When we write down (31) explicitly, we obtain

$$\mathcal{L}_{\chi_i^0}^\mathrm{mass} = -\frac{1}{2} (\Phi^0)^\mathrm{T} \mathcal{M}_N \Phi^0 + \mathrm{h.c.} \tag{32}$$

with the current basis $(\Phi^0)^\mathrm{T} = (-\mathrm{i}\lambda_B, -\mathrm{i}\lambda_A^3, \psi_{H^1}^1, \psi_{H^2}^2, \nu_{e_L}, \nu_{\mu_L}, \nu_{\tau_L})$ and

$$\mathcal{M}_N = \begin{pmatrix} 2m_1 & 0 & -\frac{1}{2}g'\upsilon_1 & \frac{1}{2}g'\upsilon_2 & -\frac{1}{2}g'\upsilon_{\tilde{\nu}_e} & -\frac{1}{2}g'\upsilon_{\tilde{\nu}_\mu} & -\frac{1}{2}g'\upsilon_{\tilde{\nu}_\tau} \\ 0 & 2m_2 & \frac{1}{2}g\upsilon_1 & -\frac{1}{2}g\upsilon_2 & \frac{1}{2}g\upsilon_{\tilde{\nu}_e} & \frac{1}{2}g\upsilon_{\tilde{\nu}_\mu} & \frac{1}{2}g\upsilon_{\tilde{\nu}_\tau} \\ -\frac{1}{2}g'\upsilon_1 & \frac{1}{2}g\upsilon_1 & 0 & -\frac{1}{2}\mu & 0 & 0 & 0 \\ \frac{1}{2}g'\upsilon_2 & -\frac{1}{2}g\upsilon_2 & -\frac{1}{2}\mu & 0 & \frac{1}{2}\epsilon_1 & \frac{1}{2}\epsilon_2 & \frac{1}{2}\epsilon_3 \\ -\frac{1}{2}g'\upsilon_{\tilde{\nu}_e} & \frac{1}{2}g\upsilon_{\tilde{\nu}_e} & 0 & \frac{1}{2}\epsilon_1 & 0 & 0 & 0 \\ -\frac{1}{2}g'\upsilon_{\tilde{\nu}_\mu} & \frac{1}{2}g\upsilon_{\tilde{\nu}_\mu} & 0 & \frac{1}{2}\epsilon_2 & 0 & 0 & 0 \\ -\frac{1}{2}g'\upsilon_{\tilde{\nu}_\tau} & \frac{1}{2}g\upsilon_{\tilde{\nu}_\tau} & 0 & \frac{1}{2}\epsilon_3 & 0 & 0 & 0 \end{pmatrix}. \tag{33}$$

The mixing is formulated as

$$\begin{aligned} -\mathrm{i}\lambda_B &= Z_N^{1i} \tilde{\chi}_i^0, & -\mathrm{i}\lambda_A^3 &= Z_N^{2i} \tilde{\chi}_i^0, \\ \psi_{H_1^1} &= Z_N^{3i} \tilde{\chi}_i^0, & \psi_{H_2^2} &= Z_N^{4i} \tilde{\chi}_i^0, \\ \nu_{e_L} &= Z_N^{5i} \tilde{\chi}_i^0, & \nu_{\mu_L} &= Z_N^{6i} \tilde{\chi}_i^0, \\ \nu_{\tau_L} &= Z_N^{7i} \tilde{\chi}_i^0 \end{aligned} \tag{34}$$

and the transformation matrix Z_N has the property

$$Z_N^\mathrm{T} \mathcal{M}_N Z_N = \mathrm{diag}(m_{\tilde{\kappa}_1^0}, m_{\tilde{\kappa}_2^0}, m_{\tilde{\kappa}_3^0}, m_{\tilde{\kappa}_4^0}, m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}). \tag{35}$$

For convenience we formulate all neutral fermions into four-component spinors as follows:

$$\nu_e = \begin{pmatrix} \tilde{\chi}_5^0 \\ \bar{\tilde{\chi}}_5^0 \end{pmatrix}, \tag{36}$$

$$\nu_\mu = \begin{pmatrix} \tilde{\chi}_6^0 \\ \bar{\tilde{\chi}}_6^0 \end{pmatrix}, \tag{37}$$

$$\nu_\tau = \begin{pmatrix} \tilde{\chi}_7^0 \\ \bar{\tilde{\chi}}_7^0 \end{pmatrix}, \tag{38}$$

$$\kappa_i^0 (i = 1, 2, 3, 4) = \begin{pmatrix} \tilde{\chi}_i^0 \\ \bar{\tilde{\chi}}_i^0 \end{pmatrix}, \tag{39}$$

From (34) we find that only one type of neutrinos obtains mass from the mixing [28]; we can assume it is the τ-neutrino. One of the stringent restrictions comes from the requirement that the mass of the τ-neutrino should be less than 20 MeV [14]. For convenience we will sometimes refer to the mixing of neutralinos and neutrinos as neutralinos.

2.3 The mixing of charginos with charged leptons

As in the case of the mixing of neutralinos and neutrinos, charginos mix with the charged leptons and form a set of charged fermions: e^-, μ^-, τ^-, $\kappa_1^\pm$, $\kappa_2^\pm$. In the current basis $\Psi^{+\mathrm{T}} = (-\mathrm{i}\lambda^+, \tilde{H}_2^1, e_R^+, \mu_R^+, \tau_R^+)$ and $\Psi^{-\mathrm{T}} = (-\mathrm{i}\lambda^-, \tilde{H}_1^2, e_L^-, \mu_L^-, \tau_L^-)$. The charged fermion mass terms in the Lagrangian can be written as [15]

$$\mathcal{L}_{\chi_i^\pm}^\mathrm{mass} = -\Psi^{-\mathrm{T}} \mathcal{M}_\mathrm{C} \Psi^+ + \mathrm{h.c.} \tag{40}$$

and the mass matrix can be written as

$$\mathcal{M}_{\rm C} = \begin{pmatrix} 2m_2 & \frac{ev_2}{\sqrt{2}S_{\rm W}} & 0 & 0 & 0 \\ \frac{ev_1}{\sqrt{2}S_{\rm W}} & \mu & \frac{l_1 v_{\tilde{\nu}_e}}{\sqrt{2}} & \frac{l_2 v_{\tilde{\nu}_\mu}}{\sqrt{2}} & \frac{l_3 v_{\tilde{\nu}_\tau}}{\sqrt{2}} \\ \frac{ev_{\tilde{\nu}_e}}{\sqrt{2}S_{\rm W}} & \epsilon_1 & \frac{l_1 v_1}{\sqrt{2}} & 0 & 0 \\ \frac{ev_{\tilde{\nu}_\mu}}{\sqrt{2}S_{\rm W}} & \epsilon_2 & 0 & \frac{l_2 v_1}{\sqrt{2}} & 0 \\ \frac{ev_{\tilde{\nu}_\tau}}{\sqrt{2}S_{\rm W}} & \epsilon_3 & 0 & 0 & \frac{l_3 v_1}{\sqrt{2}} \end{pmatrix}. \quad (41)$$

Here $S_{\rm W} = \sin\theta_{\rm W}$ and $\lambda^\pm = (\lambda_A^1 \mp {\rm i}\lambda_A^2)/\sqrt{2}$. Two mixing matrices Z_+, Z_- can be obtained by diagonalizing the mass matrix $\mathcal{M}_{\rm C}$, i.e. the product $(Z_+)^{\rm T}\mathcal{M}_{\rm C}Z_-$ is a diagonal matrix:

$$(Z_+)^{\rm T}\mathcal{M}_{\rm C}Z_- = \begin{pmatrix} m_{\kappa_1^-} & 0 & 0 & 0 & 0 \\ 0 & m_{\kappa_2^-} & 0 & 0 & 0 \\ 0 & 0 & m_e & 0 & 0 \\ 0 & 0 & 0 & m_\mu & 0 \\ 0 & 0 & 0 & 0 & m_\tau \end{pmatrix}. \quad (42)$$

If we write the mass eigenstates as $\tilde{\chi}$, then

$$\begin{aligned} -{\rm i}\lambda_A^\pm &= Z_\pm^{1i}\tilde{\chi}_i^\pm, \qquad \psi_{H^2}^1 = Z_+^{2i}\tilde{\chi}_i^+, \\ \psi_{H^1}^2 &= Z_-^{2i}\tilde{\chi}_i^-, \qquad e_L = Z_-^{3i}\tilde{\chi}_i^-, \\ e_R &= Z_+^{3i}\tilde{\chi}_i^+, \qquad \mu_L = Z_-^{4i}\tilde{\chi}_i^-, \\ \mu_R &= Z_+^{4i}\tilde{\chi}_i^+, \qquad \tau_L = Z_-^{5i}\tilde{\chi}_i^-, \\ \tau_R &= Z_+^{5i}\tilde{\chi}_i^+. \end{aligned} \quad (43)$$

The four-component fermions are defined as

$$\kappa_i^+(i=1,2,3,4,5) = \begin{pmatrix} \tilde{\chi}_i^+ \\ \bar{\tilde{\chi}}_i^- \end{pmatrix}, \quad (44)$$

where $\kappa_1^\pm$, $\kappa_2^\pm$ are the usual charginos and $\kappa_i^\pm$ $(i=3,4,5)$ correspond to e, μ and τ leptons respectively. For convenience we will sometimes refer to the mixing of charginos with charged leptons as charginos.

With the above analyses we have arrived at the mass spectrum of the neutralino–neutrinos, chargino–charged leptons, neutral Higgs–sneutrinos and charged Higgs–charged sleptons. The interaction vertices are also important; thus in the next section we will give the Feynman rules, which are different from those of the MSSM with R-parity.

3 Feynman rules for the R-parity violating interaction

We have discussed the mass spectrum of the MSSM with bilinear R-parity violation. Now we are discussing the Feynman rules for the model, which are different from those in MSSM with R-parity. We are working in the 't Hooft–Feynman gauge [16], which has the following gauge fixed terms:

$$\begin{aligned} \mathcal{L}_{\rm GF} = &-\frac{1}{2\xi}\Big(\partial^\mu A_\mu^3 + \xi M_Z C_{\rm W} H_6^0\Big)^2 \\ &-\frac{1}{2\xi}\Big(\partial^\mu B_\mu - \xi M_Z S_{\rm W} H_6^0\Big)^2 \\ &-\frac{1}{2\xi}\Big(\partial^\mu A_\mu^1 + \frac{\rm i}{\sqrt{2}}\xi M_W\Big(H_1^+ - H_1^-\Big)\Big)^2 \\ &-\frac{1}{2\xi}\Big(\partial^\mu A_\mu^2 + \frac{1}{\sqrt{2}}\xi M_W\Big(H_1^+ + H_1^-\Big)\Big)^2 \\ = &\Big\{-\frac{1}{2\xi}(\partial^\mu Z_\mu)^2 - \frac{1}{2\xi}(\partial^\mu F_\mu)^2 \\ &-\frac{1}{\xi}(\partial^\mu W_\mu^+)(\partial^\mu W_\mu^-)\Big\} - \Big\{M_Z H_6^0 \partial^\mu Z_\mu \\ &+{\rm i}M_W\Big(H_1^+\partial^\mu W_\mu^- - H_1^-\partial^\mu W_\mu^+\Big)\Big\} \\ &-\Big\{\frac{1}{2}\xi M_Z^2 H_6^{0^2} - \xi M_W^2 H_1^+ H_1^-\Big\}, \end{aligned} \quad (45)$$

where $C_{\rm W} = \cos\theta_{\rm W}$ and H_6^0, $H_1^\pm$ were given in (21) and (29). When one inserts (45) into the interaction Lagrangian, one obtains the desired vertices for the Higgs bosons. If CP is conserved, i.e. we assume that the relevant parameters are real, one finds (by analyzing the $H_i^0 f\bar{f}$ couplings) that $H_1^0, H_2^0, H_3^0, H_4^0, H_5^0$ are scalars and $H_6^0, H_7^0, H_8^0, H_9^0, H_{10}^0$ are pseudoscalars.

3.1 Feynman rules for Higgs (slepton)–gauge boson interactions

Let us compute the vertices of Higgs (slepton)–gauge bosons in the model. The original interaction terms of Higgs bosons and gauge bosons are given as

$$\begin{aligned} \mathcal{L}_{\rm int}^1 = &-\sum_I(\mathcal{D}_\mu\tilde{L}^{I\dagger}\mathcal{D}^\mu\tilde{L}^I - \mathcal{D}_\mu\tilde{R}^{I*}\mathcal{D}^\mu\tilde{R}^I) \\ &-\mathcal{D}_\mu H^{1\dagger}\mathcal{D}^\mu H^1 - \mathcal{D}_\mu H^{2\dagger}\mathcal{D}^\mu H^2 \\ = &\sum_I\Big\{\Big[{\rm i}\tilde{L}^{I\dagger}\Big(g\frac{\tau^i}{2}A_\mu^i - \frac{1}{2}g'B_\mu\Big)\partial^\mu\tilde{L}^I + {\rm h.c.}\Big] \\ &-\tilde{L}^{I\dagger}\Big(g\frac{\tau^i}{2}A_\mu^i - \frac{1}{2}g'B_\mu\Big)\Big(g\frac{\tau^j}{2}A^{j\mu} - \frac{1}{2}g'B^\mu\Big)\tilde{L}^I \\ &+\Big({\rm i}g'B_\mu\tilde{R}^{I*}\partial^\mu\tilde{R}^I + {\rm h.c.}\Big) - g'^2\tilde{R}^{I*}\tilde{R}^I B_\mu B^\mu\Big\} \\ &+\Big\{H^{1\dagger}\Big(g\frac{\tau^i}{2}A_\mu^i - \frac{1}{2}g'B_\mu\Big)\partial^\mu H^1 + {\rm h.c.}\Big\} \\ &-H^{1\dagger}\Big(g\frac{\tau^i}{2}A_\mu^i - \frac{1}{2}g'B_\mu\Big)\Big(g\frac{\tau^j}{2}A^{j\mu} - \frac{1}{2}g'B^\mu\Big)H^1 \\ &+\Big\{H^{2\dagger}\Big(g\frac{\tau^i}{2}A_\mu^i - \frac{1}{2}g'B_\mu\Big)\partial^\mu H^2 + {\rm h.c.}\Big\} \\ &-H^{2\dagger}\Big(g\frac{\tau^i}{2}A_\mu^i - \frac{1}{2}g'B_\mu\Big)\Big(g\frac{\tau^j}{2}A^{j\mu} - \frac{1}{2}g'B^\mu\Big)H^2 \\ = &\mathcal{L}_{SSV} + \mathcal{L}_{SVV} + \mathcal{L}_{SSVV}. \end{aligned} \quad (46)$$

Here $\mathcal{L}_{SSV}$, $\mathcal{L}_{SVV}$ and $\mathcal{L}_{SSVV}$ represent the interactions in the physical basis. Thus we have

$$\begin{aligned}
\mathcal{L}_{SSV} =& \frac{\mathrm{i}}{2}\sqrt{g^2+g'^2}Z_\mu\Big\{\partial^\mu\phi_1^0\chi_1^0-\phi_1^0\partial^\mu\chi_1^0-\partial^\mu\phi_2^0\chi_2^0 \\
&+\phi_2^0\partial^\mu\chi_2^0+\sum_I\Big(\partial^\mu\phi_{\tilde\nu_I}^0\chi_{\tilde\nu_I}^0-\phi_{\tilde\nu_I}^0\partial^\mu\chi_{\tilde\nu_I}^0\Big)\Big\} \\
&+\frac{1}{2}g\Big\{W_\mu^+\Big[\chi_1^0\partial^\mu H_2^1-\partial^\mu\chi_1^0H_2^1-\chi_2^0\partial^\mu H_1^{2*}+\partial^\mu\chi_2^0H_1^{2*} \\
&+\sum_I\chi_{\tilde\nu_I}^0\partial^\mu\tilde L_2^I-\partial^\mu\chi_{\tilde\nu_I}^0\tilde L_2^I\Big]+\mathrm{h.c.}\Big\} \\
&+\frac{\mathrm{i}}{2}g\Big\{W_\mu^+\Big[\phi_1^0\partial^\mu H_2^{1*}-\partial^\mu\phi_1^0H_2^1+\phi_2^0\partial^\mu H_1^{2*}-\partial^\mu\phi_2^0H_1^{2*} \\
&+\sum_I(\phi_{\tilde\nu_I}^0\partial^\mu\tilde L_2^I-\partial^\mu\phi_{\tilde\nu_I}^0\tilde L_2^I)\Big]-\mathrm{h.c.}\Big\} \\
&+\Big\{\frac{1}{2}\sqrt{g^2+g'^2}\Big(\cos2\theta_\mathrm{W}Z_\mu-\sin2\theta_\mathrm{W}A_\mu\Big) \\
&\times\Big[\sum_I\Big(\tilde L_2^{I*}\partial^\mu\tilde L_2^I-\partial^\mu\tilde L_2^{I*}\tilde L_2^I\Big)-H_1^{2*}\partial^\mu H_1^2 \\
&+\partial^\mu H_1^{2*}H_1^2+H_2^{1*}\partial^\mu H_2^1-\partial^\mu H_2^{1*}H_2^1\Big]+\Big(2\sin^2\theta_\mathrm{W}Z_\mu \\
&+2\sin\theta_\mathrm{W}\cos\theta_WA_\mu\Big)\Big[\sum_I\Big(\tilde R^{I*}\partial^\mu\tilde R^I-\partial^\mu\tilde R^{I*}\tilde R^I\Big)\Big]\Big\} \\
=& \frac{\mathrm{i}}{2}\sqrt{g^2+g'^2}C_\mathrm{eo}^{ij}\Big(\partial^\mu H_{5+i}^0H_j^0-H_{5+i}^0\partial^\mu H_j^0\Big)Z_\mu \\
&+\Big\{\frac{1}{2}gC_\mathrm{ec}^{ij}\Big(H_i^0\partial^\mu H_j^--\partial^\mu H_i^0H_j^-\Big)W_\mu^++\mathrm{h.c.}\Big\} \\
&+\Big\{\frac{\mathrm{i}}{2}gC_\mathrm{co}^{ij}\Big(H_{5+i}^0\partial^\mu H_j^--\partial^\mu H_{5+i}^0H_j^-\Big)W_\mu^++\mathrm{h.c.}\Big\} \\
&+\Big\{\frac{1}{2}\sqrt{g^2+g'^2}\Big[\Big(\cos2\theta_\mathrm{W}\delta^{ij}-C_\mathrm{c}^{ij}\Big) \\
&\times Z_\mu\Big(H_i^-\partial^\mu H_j^+-\partial^\mu H_i^-H_j^+\Big) \\
&-\sin2\theta_\mathrm{W}A_\mu\Big(H_i^-\partial^\mu H_i^+-\partial^\mu H_i^-H_i^+\Big)\Big]\Big\}, \qquad (47)
\end{aligned}$$

with

$$\begin{aligned}
C_\mathrm{eo}^{ij} &= \sum_{\alpha=1}^5 Z_\mathrm{odd}^{i,\alpha}Z_\mathrm{even}^{j,\alpha}-2Z_\mathrm{odd}^{i,2}Z_\mathrm{even}^{j,2}, \\
C_\mathrm{ec}^{ij} &= \sum_{\alpha=1}^5 Z_\mathrm{even}^{i,\alpha}Z_\mathrm{c}^{j,\alpha}-2Z_\mathrm{even}^{i,2}Z_\mathrm{c}^{j,2}, \\
C_\mathrm{co}^{ij} &= \sum_{\alpha=1}^5 Z_\mathrm{odd}^{i,\alpha}Z_\mathrm{c}^{j,\alpha}-2Z_\mathrm{odd}^{i,2}Z_\mathrm{c}^{j,2},
\end{aligned}$$

$$C_\mathrm{c}^{ij}=\sum_{\alpha=6}^8 Z_\mathrm{c}^{i,\alpha}Z_\mathrm{c}^{j,\alpha}, \qquad (48)$$

where the transformation matrices Z_even, Z_odd and Z_c are defined in Sect. 2:

$$\begin{aligned}
\mathcal{L}_{SVV} =& \frac{g^2+g'^2}{4}\Big(\upsilon_1\chi_1^0+\upsilon_2\chi_2^0+\sum_I\upsilon_{\tilde\nu_I}\chi_{\tilde\nu_I}^0\Big) \\
&\times\Big(Z_\mu Z^\mu+2\cos^2\theta_\mathrm{W}W_\mu^-W_+^\mu\Big) \\
&+\Big\{\frac{g^2+g'^2}{4}\Big[\cos\theta_\mathrm{W}\Big(-1+\cos2\theta_\mathrm{W}\Big) \\
&\times Z_\mu W_+^\mu\Big(\upsilon_1H_2^1-\upsilon_2H_1^{2*}+\sum_I\upsilon_{\tilde\nu_I}\tilde L_2^I\Big) \\
&-\cos\theta_\mathrm{W}\sin2\theta_\mathrm{W}A_\mu W^{+\mu}\Big(\upsilon_1H_2^1-\upsilon_2H_1^{2*} \\
&+\sum_I\upsilon_{\tilde\nu_I}\tilde L_2^I\Big)\Big]+\mathrm{h.c.}\Big\} \\
=& \frac{g^2+g'^2}{4}C_\mathrm{even}^i\Big(H_i^0Z_\mu Z^\mu \\
&+2\cos^2\theta_\mathrm{W}H_i^0W_\mu^-W^{+\mu}\Big)-\frac{g^2+g'^2}{2}S_\mathrm{W}C_\mathrm{W}\upsilon \\
&\times\Big[S_\mathrm{W}Z_\mu W^{+\mu}H_1^- \\
&+C_\mathrm{W}A_\mu W^{+\mu}H_1^-+\mathrm{h.c.}\Big], \qquad (49)
\end{aligned}$$

with

$$C_\mathrm{even}^i=Z_\mathrm{even}^{i,1}\upsilon_1+Z_\mathrm{even}^{i,2}\upsilon_2+\sum_I Z_\mathrm{even}^{i,I+2}\upsilon_{\tilde\nu_I}. \qquad (50)$$

$\mathcal{L}_{SSVV}$ is given as

$$\begin{aligned}
\mathcal{L}_{SSVV} =& -\frac{g^2+g'^2}{4}\Big[\frac{1}{2}\Big(\chi_1^0\chi_1^0 \\
&+\chi_2^0\chi_2^0+\sum_I\chi_{\tilde\nu_I}^0\chi_{\tilde\nu_I}^0\Big)Z_\mu Z^\mu \\
&+\cos^2\theta_\mathrm{W}\Big(\chi_1^0\chi_1^0+\chi_2^0\chi_2^0+\sum_I\chi_{\tilde\nu_I}^0\chi_{\tilde\nu_I}^0\Big)W_\mu^-W^{+\mu}\Big] \\
&-\frac{g^2+g'^2}{4}\Big[\frac{1}{2}\Big(\phi_1^0\phi_1^0+\phi_2^0\phi_2^0+\sum_I\phi_{\tilde\nu_I}^0\phi_{\tilde\nu_I}^0\Big)Z_\mu Z^\mu \\
&+\cos^2\theta_\mathrm{W}\Big(\phi_1^0\phi_1^0+\phi_2^0\phi_2^0+\phi_{\tilde\nu_I}^0\phi_{\tilde\nu_I}^0\Big)W_\mu^-W^{+\mu}\Big] \\
&-\frac{g^2+g'^2}{4}\cos\theta_\mathrm{W}\Big[\Big(-1+\cos2\theta_\mathrm{W}\Big)Z_\mu W^{+\mu} \\
&\times\Big(\chi_1^0H_2^1-\chi_2^0H_1^{2*}+\sum_I\chi_{\tilde\nu_I}^0\tilde L_2^I\Big) \\
&-\cos\theta_\mathrm{W}\sin2\theta_\mathrm{W}A_\mu W^{+\mu} \\
&\times\Big(\chi_1^0H_2^1-\chi_2^0H_1^{2*}+\sum_I\chi_{\tilde\nu_I}^0\tilde L_2^I\Big)+\mathrm{h.c.}\Big]
\end{aligned}$$

$$
\begin{aligned}
&+\frac{\mathrm{i}(g^2+g'^2)}{4}\cos\theta_{\mathrm{W}}\Bigg[\Big(-1+\cos 2\theta_{\mathrm{W}}\Big)Z_\mu W^{+\mu}\\
&\times\Big(\phi_1^0 H_2^1-\phi_2^0 H_1^{2*}+\sum_I \phi_{\tilde\nu_I}^0\tilde L_2^I\Big)\\
&-\cos\theta_{\mathrm{W}}\sin 2\theta_{\mathrm{W}} A_\mu W^{+\mu}\Big(\phi_1^0 H_2^1\\
&-\phi_2^0 H_1^{2*}+\sum_I \phi_{\tilde\nu_I}^0\tilde L_2^I\Big)+\mathrm{h.c.}\Bigg]\\
&-\frac{1}{4}(g^2+g'^2)\Bigg[\sin^2 2\theta_{\mathrm{W}} A_\mu A^\mu\\
&\times\Big(H_2^{1*}H_2^1+H_1^{2*}H_1^2+\sum_I \tilde L_2^{I*}\tilde L_2^I\Big)\\
&+\cos^2 2\theta_{\mathrm{W}} Z_\mu Z^\mu\Big(H_2^{1*}H_2^1+H_1^{2*}H_1^2+\sum_I \tilde L_2^{I*}\tilde L_2^I\Big)\\
&-\sin 4\theta_{\mathrm{W}} Z_\mu A^\mu\Big(H_2^{1*}H_2^1+H_1^{2*}H_1^2+\sum_I \tilde L_2^{I*}\tilde L_2^I\Big)\\
&+2\cos^2\theta_{\mathrm{W}}\Big(H_2^{1*}H_2^1+H_1^{2*}H_1^2+\sum_I \tilde L_2^{I*}\tilde L_2^I\Big)\Bigg]\\
&-\sum_I g'^2\tilde R^{I*}\tilde R^I B_\mu B^\mu\\
=&-\frac{1}{4}(g^2+g'^2)\Big(\frac{1}{2}H_i^0 H_i^0 Z_\mu Z^\mu\\
&+\cos^2\theta_{\mathrm{W}} H_i^0 H_i^0 W_\mu^- W^{+\mu}\Big)\\
&-\frac{1}{4}(g^2+g'^2)\Big(\frac{1}{2}H_{5+i}^0 H_{5+i}^0 Z_\mu Z^\mu\\
&+\cos^2\theta_{\mathrm{W}} H_{5+i}^0 H_{5+i}^0 W_\mu^- W^{+\mu}\Big)\\
&+\frac{1}{4}(g^2+g'^2)\sin 2\theta_{\mathrm{W}}\Bigg\{C_{\mathrm{ec}}^{ij}\Big[\sin\theta_{\mathrm{W}} H_i^0 Z_\mu W^{+\mu}H_j^-\\
&+\cos\theta_{\mathrm{W}} H_i^0 A_\mu W^{+\mu}H_j^-\Big]+\mathrm{h.c.}\Bigg\}\\
&-\frac{\mathrm{i}}{4}(g^2+g'^2)\sin 2\theta_{\mathrm{W}}\Bigg\{C_{\mathrm{co}}^{ij}\Big[\sin\theta_{\mathrm{W}} H_{5+i}^0 Z_\mu W^{+\mu}H_j^-\\
&+\cos\theta_{\mathrm{W}} H_{5+i}^0 A_\mu W^{+\mu}H_i^-\Big]-\mathrm{h.c.}\Bigg\}\\
&-\frac{1}{4}(g^2+g'^2)\Bigg\{2\cos^2\theta_{\mathrm{W}}\Big(\delta_{ij}-C_{\mathrm{c}}^{ij}\Big)H_i^- H_j^+ W_\mu^- W^{+\mu}\\
&+\Big[\cos^2 2\theta_{\mathrm{W}}\delta_{ij}\\
&-C_{\mathrm{c}}^{ij}\Big(4\sin^3\theta_{\mathrm{W}}-\cos^2 2\theta_{\mathrm{W}}\Big)\Big]H_i^- H_j^+ Z_\mu Z^\mu\\
&+\sin^2 2\theta_{\mathrm{W}}\delta_{ij}H_i^- H_j^+ A_\mu A^\mu+\Big[\sin 4\theta_{\mathrm{W}}\delta_{ij} \qquad (51)\\
&-C_{\mathrm{c}}^{ij}\Big(\sin 4\theta_{\mathrm{W}}+8\sin^2\theta_{\mathrm{W}}\cos\theta_{\mathrm{W}}\Big)\Big]Z_\mu A^\mu H_i^- H_j^+\Bigg\},
\end{aligned}
$$

where the C_{eo}^{ij}, C_{co}^{ij} and C_{c}^{ij} are defined in (48). The relevant Feynman rules are summarized in Figs. 1, 2, 3 and 4. We will emphasize some of their features. First, the presence of the vertices $Z_\mu H_i^0 H_{5+j}^0$ $(i,j=1,2,3,4,5)$ and the forbiddance of the vertices $Z_\mu H_i^0 H_j^0$ and $Z_\mu H_{5+i}^0 H_{5+j}^0$ $(i,j=1,2,3,4,5)$ couplings are determined by the CP nature. Second, besides the $W_\mu^+ Z^\mu H_1^-$ interaction (H_1^- is just the charged Goldstone boson), there are no vertices $W_\mu^+ Z^\mu H_i^-$ $(i=2,3,4,5,6,7,8)$ at the tree level. This is the same as in the MSSM with R-parity and in general two-Higgs doublet models [17].

3.2 Self-couplings of the Higgs bosons (sleptons)

It is straightforward to insert (18), (21), (22), (29) and (30) into (6) to obtain the desired interaction terms. As we did in the case of the gauge–Higgs (slepton) bosons interaction, we split the Lagrangian:

$$\mathcal{L}_{\mathrm{int}}^S=\mathcal{L}_{SSS}+\mathcal{L}_{SSSS}, \qquad (52)$$

where $\mathcal{L}_{SSS}$ represents trilinear coupling terms and $\mathcal{L}_{SSSS}$ represents four scalar boson coupling terms. The trilinear part is most interesting. If the masses of the scalars are appropriate, the decays of one Higgs boson into two other Higgs bosons may occur. After tedious computation we arrive at

$$
\begin{aligned}
\mathcal{L}_{SSS}=&-\frac{g^2+g'^2}{8}A_{\mathrm{even}}^{ij}B_{\mathrm{even}}^k H_i^0 H_j^0 H_k^0\\
&-\frac{g^2+g'^2}{8}A_{\mathrm{odd}}^{ij}B_{\mathrm{even}}^k H_{5+i}^0 H_{5+j}^0 H_k^0\\
&-A_{\mathrm{ec}}^{kij}H_k^0 H_i^- H_j^+ +\mathrm{i}A_{\mathrm{oc}}^{kij}H_{5+k}^0 H_i^- H_j^+ \qquad (53)
\end{aligned}
$$

and

$$
\begin{aligned}
\mathcal{L}_{SSSS}=&-\frac{g^2+g'^2}{32}A_{\mathrm{even}}^{ij}A_{\mathrm{even}}^{kl}H_i^0 H_j^0 H_k^0 H_l^0\\
&-\frac{g^2+g'^2}{32}A_{\mathrm{odd}}^{ij}A_{\mathrm{odd}}^{kl}H_{5+i}^0 H_{5+j}^0 H_{5+k}^0 H_{5+l}^0\\
&-\frac{g^2+g'^2}{16}A_{\mathrm{even}}^{ij}A_{\mathrm{odd}}^{kl}H_i^0 H_j^0 H_{5+k}^0 H_{5+l}^0\\
&-\mathcal{A}_{\mathrm{ec}}^{klij}H_k^0 H_l^0 H_i^- H_j^+\\
&-\mathcal{A}_{\mathrm{oc}}^{klij}H_{5+k}^0 H_{5+l}^0 H_i^- H_j^+ -\mathrm{i}\mathcal{A}_{\mathrm{eoc}}^{klij}H_k^0 H_{5+l}^0 H_i^- H_j^+\\
&-\mathcal{A}_{\mathrm{cc}}^{klij}H_k^- H_l^+ H_i^- H_j^+, \qquad (54)
\end{aligned}
$$

with

$$
\begin{aligned}
A_{\mathrm{even}}^{ij}&=\sum_{\alpha=1}^{5}Z_{\mathrm{even}}^{i,\alpha}Z_{\mathrm{even}}^{j,\alpha}-2Z_{\mathrm{even}}^{i,2}Z_{\mathrm{even}}^{j,2},\\
A_{\mathrm{odd}}^{ij}&=\sum_{\alpha=1}^{5}Z_{\mathrm{odd}}^{i,\alpha}Z_{\mathrm{odd}}^{j,\alpha}-2Z_{\mathrm{odd}}^{i,2}Z_{\mathrm{odd}}^{j,2},\\
B_{\mathrm{even}}^{i}&=\upsilon_1 Z_{\mathrm{even}}^{i,1}-\upsilon_2 Z_{\mathrm{even}}^{i,2}+\sum_I \upsilon_{\tilde\nu_I}Z_{\mathrm{even}}^{i,I+2}. \qquad (55)
\end{aligned}
$$

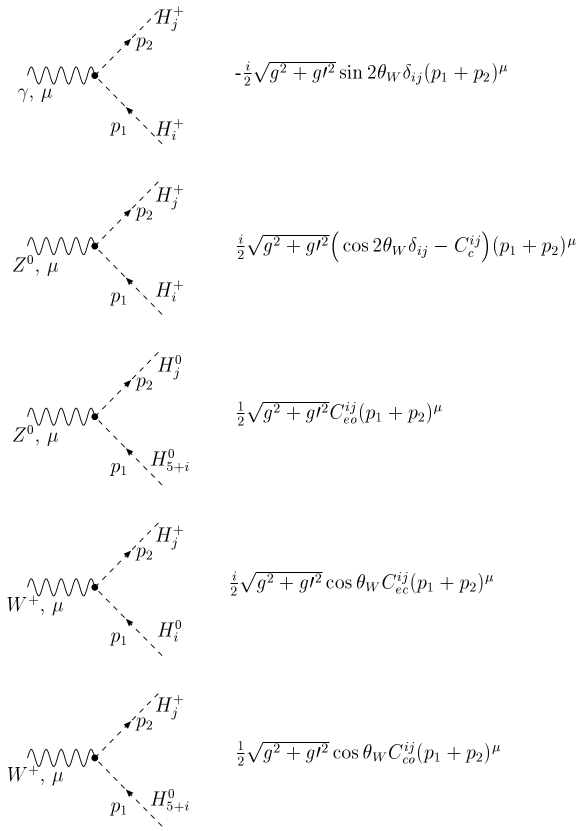


Fig. 1. Feynman rules for SSV vertices. The direction of momentum is indicated above

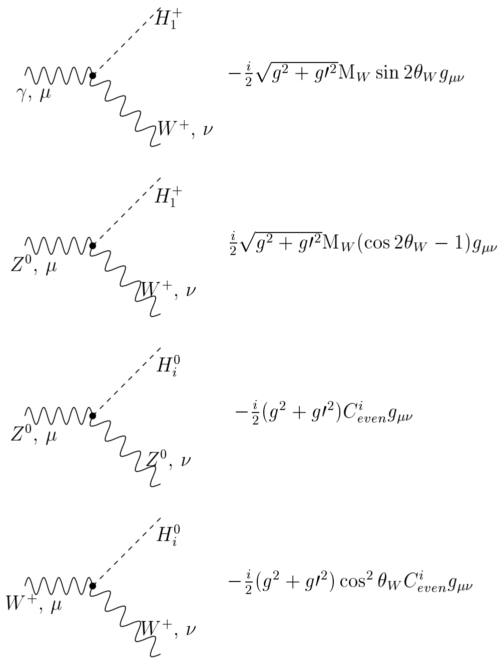
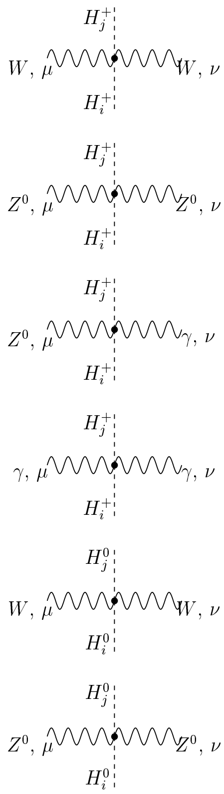


Fig. 2. Feynman rules for SVV vertices



$-\frac{i}{2}(g^2+g\prime^2)\cos^2\theta_W\Big(\delta_{ij}-C_c^{ij}\Big)g_{\mu\nu}$

$-\frac{i}{2}(g^2+g\prime^2)\Big(\cos^2 2\theta_W\delta_{ij}-C_c^{ij}\Big(4\sin^3\theta_W-\cos^2 2\theta_W\Big)\Big)g_{\mu\nu}$

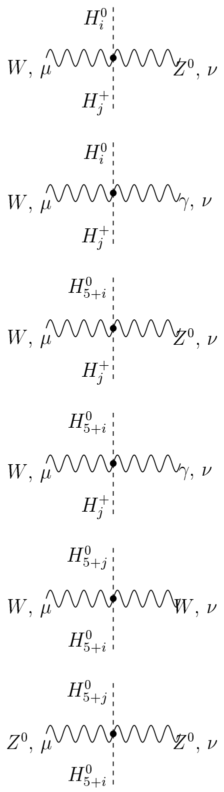
$-\frac{i}{4}(g^2+g\prime^2)\Big[\sin 4\theta_W\delta_{ij}-C_c^{ij}\Big(\sin 4\theta_W+8\sin^2\theta_W\cos\theta_W\Big)\Big]g_{\mu\nu}$

$-\frac{i}{2}(g^2+g\prime^2)\sin^2 2\theta_W\delta_{ij}g_{\mu\nu}$

$-\frac{i}{2}(g^2+g\prime^2)\cos^2\theta_W\delta_{ij}g_{\mu\nu}$

$-\frac{i}{2}(g^2+g\prime^2)\delta_{ij}g_{\mu\nu}$

Fig. 3. Feynman rules for $SSVV$ vertices. Part (I)



$\frac{i}{2}(g^2+g\prime^2)\cos\theta_W\sin^2\theta_W C_{ec}^{ij}g_{\mu\nu}$

$-\frac{i}{2}(g^2+g\prime^2)\cos^2\theta_W\sin\theta_W C_{ec}^{ij}g_{\mu\nu}$

$-\frac{1}{2}(g^2+g\prime^2)\cos\theta_W\sin^2\theta_W C_{co}^{ij}g_{\mu\nu}$

$\frac{1}{2}(g^2+g\prime^2)\cos^2\theta_W\sin\theta_W C_{co}^{ij}g_{\mu\nu}$

$-\frac{i}{2}(g^2+g\prime^2)\cos^2\theta_W\delta_{ij}g_{\mu\nu}$

$-\frac{i}{2}(g^2+g\prime^2)\delta_{ij}g_{\mu\nu}$

Fig. 4. Feynman rules for $SSVV$ vertices. Part (II)

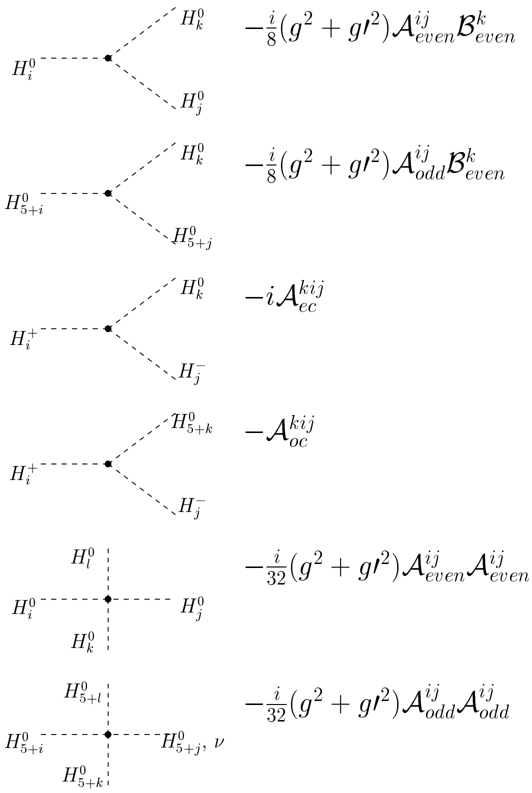


Fig. 5. Feynman rules for the self-coupling of Higgs. Part (I)

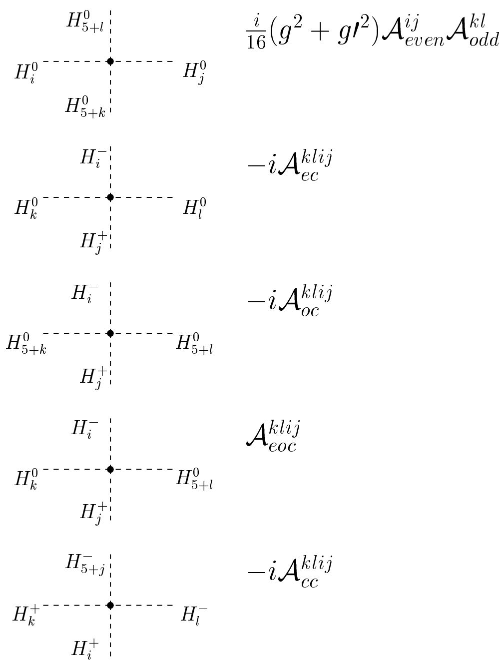


Fig. 6. Feynman rules for the self-coupling of Higgs. Part (II)

The definitions of $A_{\rm ec}^{kij}$, $A_{\rm oc}^{kij}$, $\mathcal{A}_{\rm ec}^{klij}$, $\mathcal{A}_{\rm oc}^{klij}$, $\mathcal{A}_{\rm eoc}^{klij}$ and $\mathcal{A}_{\rm cc}^{ijkl}$ can be found in Appendix C. The Feynman rules are summarized in Figs. 5 and 6. Note that the lepton number violation has led to very complicated form for the $\mathcal{L}_{SSS}$ and $\mathcal{L}_{\rm SSSS}$.

3.3 The couplings of Higgs to charginos (charged leptons) and neutralinos (neutrinos)

In this subsection we compute the interactions of the Higgs bosons with the supersymmetric partners of the gauge and Higgs bosons (the gauginos and higgsinos). After spontaneous breaking of the gauge symmetry $SU(2)\times U(1)$, the gauginos, higgsinos and leptons with the same electric charge will mix as we have described in Sect. 2. Let us now compute interesting interactions $S\tilde{\kappa}_i^0\tilde{\kappa}_j^0$ (Higgs–neutralinos–neutralinos interactions) etc.

The original interactions (in two-component notations) are [18]

$$\begin{aligned}
\mathcal{L}_{S\kappa\kappa} =& \,\mathrm{i}\sqrt{2}g\Big(H^{1\dagger}\frac{\tau^i}{2}\lambda_A^i\psi_{H^1} - \bar{\psi}_{H^1}\frac{\tau^i}{2}\bar{\lambda}_A^i H^1\Big)\\
&-\mathrm{i}\sqrt{2}g'\Big(\frac{1}{2}H^{1\dagger}\psi_{H^1}\lambda_B - \frac{1}{2}\bar{\lambda}_B\bar{\psi}_{H^1}H^1\Big)\\
&+\mathrm{i}\sqrt{2}g\Big(H^{2\dagger}\frac{\tau^i}{2}\lambda_A^i\psi_{H^2} - \bar{\psi}_{H^2}\frac{\tau^i}{2}\bar{\lambda}_A^i H^2\Big)\\
&+\mathrm{i}\sqrt{2}g'\Big(\frac{1}{2}H^{2\dagger}\psi_{H^2}\lambda_B - \frac{1}{2}\bar{\lambda}_B\bar{\psi}_{H^2}H^2\Big)\\
&+\mathrm{i}\sqrt{2}\tilde{L}^{I\dagger}\Big(g\frac{\tau^i}{2}\lambda_A^i\psi_{L^I} - \frac{1}{2}g'\lambda_B\psi_{L^I}\Big)\\
&-\mathrm{i}\sqrt{2}\tilde{L}^{I}\Big(g\frac{\tau^i}{2}\bar{\lambda}_A^i\bar{\psi}_{L^I} - \frac{1}{2}g'\bar{\lambda}_B\bar{\psi}_{L^I}\Big)\\
&+\mathrm{i}\sqrt{2}g'\tilde{R}^{I\dagger}\lambda_B\psi_{R^I} - \mathrm{i}\sqrt{2}g'\tilde{R}^{I}\bar{\lambda}_B\bar{\psi}_{R^I}\\
&-\frac{1}{2}l_I\varepsilon_{ij}\Big(\psi_{H^1}^i\psi_{L^I}^j\tilde{R}^I\\
&+\psi_{H^1}^i\psi_{R^I}\tilde{R}_j^I + \psi_{R^I}\psi_{L^I}^j H_i^1 + \mathrm{h.c.}\Big). \qquad (56)
\end{aligned}$$

Now we sketch the derivation for the vertices, such as $S\tilde{\kappa}_i^0\tilde{\kappa}_j^0$. We start with (56) and we first convert the parts from two-component notations into four-component notations. Then, when the spinor fields defined by (36), (37), (38), (39) and (44) are used, we find

$$\begin{aligned}
\mathcal{L}_{S\kappa\kappa} =& \,\frac{\sqrt{g^2+g'^2}}{2}\Big[C_{snn}^{ij}H_i^0\bar{\kappa}_j^0P_L\kappa_m^0 + C_{snn}^{ij*}H_i^0\bar{\kappa}_j^0P_R\kappa_m^0\Big]\\
&+\frac{g}{\sqrt{2}}\Big[C_{skk}^{ij}H_i^0\bar{\kappa}_m^+P_L\kappa_j^+ + C_{skk}^{ij*}H_i^0\bar{\kappa}_j^+P_R\kappa_m^+\Big]\\
&+\mathrm{i}\frac{\sqrt{g^2+g'^2}}{2}\Big[C_{\rm onn}^{ij}H_{5+i}^0\bar{\kappa}_j^0P_R\kappa_m^0\\
&-C_{\rm onn}^{ij*}H_{5+i}^0\bar{\kappa}_m^0P_L\kappa_j^0\Big]\\
&+i\frac{g}{\sqrt{2}}\Big[C_{\rm okk}^{ij}H_{5+i}^0\bar{\kappa}_m^+P_L\kappa_j^+ - C_{\rm okk}^{ij*}H_{5+i}^0\bar{\kappa}_m^+P_R\kappa_j^+\Big]\\
&+\sqrt{g^2+g'^2}\Big[C_{\rm Lnk}^{ij}\bar{\kappa}_j^+P_L\kappa_m^0H_i^+\\
&-C_{\rm Rnk}^{ij}\bar{\kappa}_j^+P_R\kappa_m^0H_i^+\Big]. \qquad (57)
\end{aligned}$$

The definitions of $C_{\rm snn}^{ij}$, $C_{\rm Lnk}^{ij}$, $C_{\rm Rnk}^{ij}$ and C_{skk}^{ij} are as follows:

$$\begin{aligned}
C_{\rm snn}^{ij} =& \Big(\cos\theta_{\rm W}Z_N^{j,2} - \sin\theta_{\rm W}Z_N^{j,1}\Big)\\
&\times\Big(\sum_{\alpha=1}^{5}Z_{\rm even}^{i,\alpha}Z_N^{m,2+\alpha} - 2Z_{\rm even}^{i,2}Z_N^{m,4}\Big),\\
C_{\rm skk}^{ij} =& \Big(Z_{\rm even}^{i,1}Z_+^{j,1}Z_-^{m,2}\\
&+Z_{\rm even}^{i,2}Z_+^{j,2}Z_-^{m,1} + \sum_{\alpha=3}^{5}Z_{\rm even}^{i,\alpha}Z_+^{j,1}Z_-^{m,\alpha}\Big)\\
&+\frac{1}{2g}\sum_{I=1}^{3}l_I\Big(Z_{\rm even}^{i,I+2}Z_+^{j,I+2}Z_-^{m,2}\\
&-Z_{\rm even}^{i,1}Z_+^{j,I+2}Z_-^{m,I+2}\Big),\\
C_{\rm onn}^{ij} =& \Big(\cos\theta_{\rm W}Z_N^{j,2} - \sin\theta_{\rm W}Z_N^{j,1}\Big)\\
&\times\Big(\sum_{\alpha=1}^{5}Z_{\rm odd}^{i,\alpha}Z_N^{m,2+\alpha} - 2Z_{\rm odd}^{i,2}Z_N^{m,4}\Big),\\
C_{\rm okk}^{ij} =& \Big(Z_{\rm odd}^{i,1}Z_+^{j,1}Z_-^{m,2} + Z_{\rm odd}^{i,2}Z_+^{j,2}Z_-^{m,1}\\
&+\sum_{\alpha=1}^{3}Z_{\rm odd}^{i,2+\alpha}Z_+^{j,1}Z_-^{m,2+\alpha}\Big)\\
&+\frac{\mathrm{i}}{2g}\sum_{I}l_I\Big(Z_{\rm odd}^{i,2+I}Z_+^{j,2+I}Z_-^{m,2}\\
&-Z_{\rm odd}^{i,1}Z_+^{j,2+I}Z_-^{m,2+I}\Big),\\
C_{\rm Lnk}^{ij} =& \Big[Z_{\rm c}^{i,1}\Big(\frac{1}{\sqrt{2}}\Big(\cos\theta_{\rm W}Z_-^{j,2}Z_N^{m,2} + \sin\theta_{\rm W}Z_-^{j,2}Z_N^{m,1}\Big)\\
&-\cos\theta_{\rm W}Z_-^{j,1}Z_N^{m,3}\Big)\\
&+\sum_{\alpha=3}^{5}Z_{\rm c}^{i,\alpha}\Big(\frac{1}{\sqrt{2}}\Big(\cos\theta_{\rm W}Z_-^{j,\alpha}Z_N^{m,2}\\
&+\sin\theta_{\rm W}Z_-^{j,\alpha}Z_N^{m,1}\Big) - \cos\theta_{\rm W}Z_-^{j,1}Z_N^{m,2+\alpha}\Big)\Big]\\
&+\frac{1}{2\sqrt{g^2+g'^2}}\sum_{I=1}^{3}l_I\Big(Z_{\rm c}^{i,5+I}Z_-^{j,2+I}Z_N^{m,3}\\
&-Z_{\rm c}^{i,5+I}Z_-^{j,2}Z_N^{m,4+I}\Big),\\
C_{\rm Rnk}^{ij} =& \Big[Z_{\rm c}^{i,2}\Big(\frac{1}{\sqrt{2}}\Big(\cos\theta_{\rm W}Z_+^{*j,2}Z_N^{*m,2}\\
&+\sin\theta_{\rm W}Z_+^{*j,2}Z_N^{*m,1}\Big) + \cos\theta_{\rm W}Z_+^{*j,1}Z_N^{*m,4}\Big)\\
&+\sqrt{2}\sin\theta_{\rm W}\sum_{I=1}^{3}Z_{\rm c}^{i,5+I}Z_+^{*j,2+I}Z_N^{*m,1}\Big]
\end{aligned}$$

$$+\frac{1}{2\sqrt{g^2+g'^2}}\sum_{I=1}^{3} l_I\Big(Z_{\rm c}^{i,2+I}Z_N^{*m,3}Z_+^{*j,2+I}$$
$$-Z_{\rm c}^{i,1}Z_N^{*m,3}Z_+^{*j,2+I}\Big). \qquad (58)$$

The project operators $P_{L,R}=(1\pm\gamma_5)/2$ and the transformation matrices $Z_\pm$, Z_N are defined in Sect. 2. The corresponding Feynman rules are summarized in Fig. 7. As for κ_i^0 being a Majorana fermion, we note the useful identity

$$\bar{\kappa}_j^0(1\pm\gamma_5)\kappa_k^0=\bar{\kappa}_k^0(1\pm\gamma_5)\kappa_j^0, \qquad (59)$$

which holds for anticommuting four-component Majorana spinors. This implies that the $H_i^0\bar{\kappa}_j^0\kappa_k^0$ interactions can be rearranged in symmetry under the interchange of j and k.

Since ν_e (e), ν_μ (μ) and ν_τ (τ) should be identified with the three lightest neutralinos (charginos) in the model, there must be some interesting phenomena relevant to them, such as κ_i^0 $(i=1,2,3,4)\to\tau H_j^+$ $(j=2,3,\cdots,8)$, κ_i^0 $(i=1,2,3,4)\to\nu_{e,\mu,\tau}H_j^0$ $(j=1,2,\cdots,5)$, if the masses are suitable. Namely, these interactions without R-parity conservation may induce interesting rare processes [20].

3.4 The couplings of gauge bosons to charginos (charged leptons) and neutralinos (neutrinos)

In this subsection we will focus on the couplings of the gauge bosons (W,Z,γ) to the charginos (charged leptons) and neutralinos (neutrinos). Since we identify the three types of charged leptons (three types of neutrinos) with the three lightest charginos (neutralinos), the restrictions relating to them from the present experiments must be considered carefully. The relevant interactions come from the following parts:

$$\mathcal{L}_{\rm int}^{\rm gcn}=-{\rm i}\bar{\lambda}_A^i\bar{\sigma}^\mu\mathcal{D}_\mu\lambda_A^i-{\rm i}\bar{\lambda}_B\bar{\sigma}^\mu\mathcal{D}_\mu\lambda_B-{\rm i}\bar{\psi}_{H^1}\bar{\sigma}^\mu\mathcal{D}_\mu\psi_{H^1}$$
$$-{\rm i}\bar{\psi}_{H^2}\bar{\sigma}^\mu\mathcal{D}_\mu\psi_{H^2}-{\rm i}\bar{\psi}_{L^I}\bar{\sigma}^\mu\mathcal{D}_\mu\psi_{L^I}$$
$$-{\rm i}\bar{\psi}_{R^I}\bar{\sigma}^\mu\mathcal{D}_\mu\psi_{R^I}, \qquad (60)$$

with

$$\mathcal{D}_\mu\lambda_A^1=\partial_\mu\lambda_A^1-gA_\mu^2\lambda_A^3+gA_\mu^3\lambda_A^2,$$
$$\mathcal{D}_\mu\lambda_A^2=\partial_\mu\lambda_A^2-gA_\mu^3\lambda_A^1+gA_\mu^1\lambda_A^3,$$
$$\mathcal{D}_\mu\lambda_A^3=\partial_\mu\lambda_A^3-gA_\mu^1\lambda_A^2+gA_\mu^2\lambda_A^1,$$
$$\mathcal{D}_\mu\lambda_B=\partial_\mu\lambda_B,$$
$$\mathcal{D}_\mu\psi_{H^1}=(\partial_\mu+{\rm i}gA_\mu^i\frac{\tau^i}{2}-\frac{\rm i}{2}g'B_\mu)\psi_{H^1},$$
$$\mathcal{D}_\mu\psi_{H^2}=(\partial_\mu+{\rm i}gA_\mu^i\frac{\tau^i}{2}+\frac{\rm i}{2}g'B_\mu)\psi_{H^2},$$
$$\mathcal{D}_\mu\psi_{L^I}=(\partial_\mu+{\rm i}gA_\mu^i\frac{\tau^i}{2}-\frac{\rm i}{2}g'B_\mu)\psi_{L^I},$$
$$\mathcal{D}_\mu\psi_{R^I}=(\partial_\mu+{\rm i}g'B_\mu)\psi_{R^I}. \qquad (61)$$

As we did in the case of the couplings in $\mathcal{L}_{S\kappa\kappa}$, we convert all spinors in (60) into four-component ones, and when we use (39) and (44), we obtain

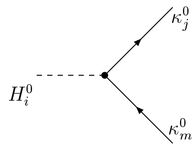
$$\mathcal{L}_{\rm int}^{\rm gcn}=\Bigg\{\sqrt{g^2+g'^2}\sin\theta_{\rm W}\cos\theta_{\rm W}A_\mu\bar{\kappa}_i^+\gamma\kappa_i^+$$
$$-\sqrt{g^2+g'^2}Z_\mu\bar{\kappa}_i^+\Bigg[\cos^2\theta_{\rm W}\delta_{ij}\gamma^\mu$$
$$+\frac{1}{2}\Big(Z_-^{*i,2}Z_-^{j,2}+\sum_{I=1}^{3}Z_-^{*i,2+I}Z_-^{j,2+I}\Big)\gamma^\mu P_R$$
$$+\Big(\frac{1}{2}Z_+^{*i,2}Z_+^{j,2}-\sum_{I=1}^{3}Z_+^{*i,2+I}Z_+^{j,2+I}\Big)\gamma^\mu P_L\Bigg]\kappa_j^+\Bigg\}$$
$$+\Bigg\{g\bar{\kappa}_j^+\Bigg[\Big(-Z_+^{*i,1}Z_N^{j,2}+\frac{1}{\sqrt{2}}Z_+^{*i,2}Z_N^{j,4}\Big)\gamma^\mu P_L$$
$$+\Big(Z_N^{*i,2}Z_-^{j,1}+\frac{1}{\sqrt{2}}\Big(Z_N^{*i,3}Z_-^{j,2}$$
$$+\sum_{I=1}^{3}Z_N^{*i,4+I}Z_-^{j,2+I}\Big)\Big)\gamma^\mu P_R\Bigg]\kappa_i^0W_\mu^+ + {\rm h.c.}\Bigg\}$$
$$+\frac{\sqrt{g^2+g'^2}}{2}\bar{\kappa}_i^0\gamma^\mu\Bigg[\frac{1}{2}\Big(Z_N^{*i,4}Z_N^{j,4}-\Big(Z_N^{*i,3}Z_N^{j,3}$$
$$+\sum_{\alpha=5}^{7}Z_N^{*i,\alpha}Z_N^{j,\alpha}\Big)\Big)P_L$$
$$-\frac{1}{2}\Big(Z_N^{*i,4}Z_N^{j,4}-\Big(Z_N^{*i,3}Z_N^{j,3}$$
$$+\sum_{\alpha=5}^{7}Z_N^{*i,\alpha}Z_N^{j,\alpha}\Big)\Big)P_R\Bigg]\kappa_j^0Z_\mu. \qquad (62)$$

The corresponding Feynman rules are summarized in Fig. 8. Before we identify the three lightest neutralinos (charginos) with the three types of neutrinos (charged leptons), we want to emphasize some features of (62):

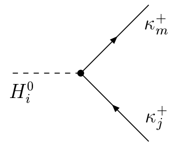
– For the γ-boson–κ–κ vertices, there is no lepton flavor changing current interaction at the tree level, which is the same as for the SM and MSSM with R-parity.
– For the tree level Z-boson–κ–κ vertices, there are lepton flavor changing current interactions. This point is different from the MSSM with R-parity.
– As in the case of the Z-boson–κ–κ vertices, there are tree level vertices, such as $W\tau\nu_e$, which are forbidden in the MSSM with R-parity.

3.5 The interactions of quarks and/or squarks with charginos (charged leptons) and/or neutralinos (neutrinos)

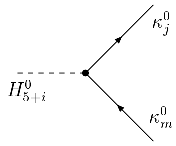
In this subsection we will give the Feynman rules for the interactions of quarks and squarks with charginos (charged leptons) and neutralinos (neutrinos), i.e. the $\tilde{Q}q\kappa_i^\pm$ vertices. Because of lepton number violation, so with mixing of neutrinos (charged leptons) and original neutralinos (charginos), the vertices may lead to interesting phenomena, thus it is interesting to write them out. There are



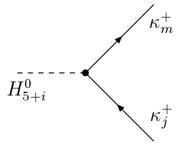
$$i\frac{\sqrt{g^2+g'^2}}{2}\Big(C^{jm}_{snn}P_L + C^{jm*}_{snn}P_R\Big)$$



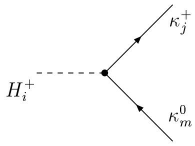
$$i\frac{g}{\sqrt{2}}\Big(C^{jm}_{skk}P_L + C^{jm*}_{skk}P_R\Big)$$



$$-\frac{\sqrt{g^2+g'^2}}{2}\Big(C^{jm}_{onn}P_L - C^{jm*}_{onn}P_R\Big)$$

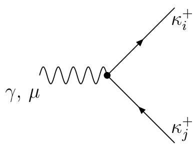


$$-\frac{g}{\sqrt{2}}\Big(C^{jm}_{okk}P_L - C^{jm*}_{okk}P_R\Big)$$

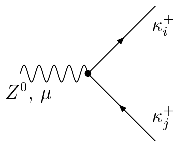


$$i\sqrt{g^2+g'^2}\Big(C^{ij}_{Lnk}P_L - C^{ij}_{Rnk}P_R\Big)$$

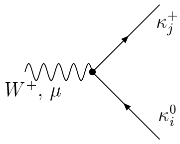
Fig. 7. Feynman rules for the coupling of Higgs with charginos or neutralinos



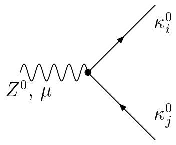
$$i\sqrt{g^2+g'^2}\cos\theta_W\sin\theta_W\gamma^\mu\delta_{ij}$$



$$i\sqrt{g^2+g'^2}\gamma^\mu[\cos^2\theta_W\delta_{ij} + (\tfrac{1}{2}Z_+^{*i,2}Z_+^{j,2} - \sum_{I=1}^{3} Z_+^{*i,2+I}Z_+^{j,2+I})P_L$$
$$+\tfrac{1}{2}(Z_-^{*i,2}Z_-^{j,2} + \sum_{I=1}^{3} Z_-^{*i,2+I}Z_-^{j,2+I})P_R]$$



$$ig\gamma^\mu[(-Z_+^{*i,1}Z_N^{j,2} + \tfrac{1}{\sqrt{2}}Z_+^{*i,2}Z_N^{j,4})P_L + (Z_N^{*i,2}Z_-^{j,1}$$
$$+\tfrac{1}{\sqrt{2}}(Z_N^{*i,3}Z_-^{j,2} + \sum_{I=1}^{3} Z_N^{*i,4+I}Z_-^{j,2+I}))P_R]$$



$$i\frac{\sqrt{g^2+g'^2}}{2}\Big[\tfrac{1}{2}\Big(Z_N^{*i,4}Z_N^{j,4} - \Big(Z_N^{*i,3}Z_N^{j,3} + \textstyle\sum_{\alpha=5}^{7} Z_N^{*i,\alpha}Z_N^{j,\alpha}\Big)\Big)P_L$$
$$-\tfrac{1}{2}\Big(Z_N^{*i,4}Z_N^{j,4} - \Big(Z_N^{*i,3}Z_N^{j,3} + \textstyle\sum_{\alpha=5}^{7} Z_N^{*i,\alpha}Z_N^{j,\alpha}\Big)\Big)P_R\Big]$$

Fig. 8. Feynman rules for the coupling of gauge bosons with charginos or neutralinos

two parts contributing to the above vertices. The first is the supersymmetric analogue of the $q\bar{q}W^{\pm}$ and $q\bar{q}Z$ interaction. The second is the supersymmetric analogue of the $q\bar{q}H$ interaction, which is proportional to the quark mass and depends on the properties of the Higgs bosons in the model. These two kinds of vertices correspond to the terms in (31).

We consider the $\bar{q}q\kappa_i^{\pm}$ interaction first. Let us write down the interaction in two-component spinors for fermions as follows:

$$\begin{aligned}\mathcal{L}_{\tilde{Q}q\kappa^{\pm}} &= \mathrm{i}g\Big(C^{IJ}\tilde{Q}_2^{I*}\lambda_A^{-}\psi_{Q_1}^J + C^{IJ*}\tilde{Q}_1^{J*}\lambda_A^{+}\psi_{Q_2}^I\Big)\\ &-\mathrm{i}g\Big(C^{IJ*}\tilde{Q}_2^{I}\bar{\lambda}_A^{-}\bar{\psi}_{Q_1}^J + C^{IJ}\tilde{Q}_1^{J}\bar{\lambda}_A^{+}\bar{\psi}_{Q_2}^I\Big)\\ &-\frac{d^I}{2}\Big(C^{IJ}\psi_{H^1}^2\psi_{Q^1}^J\tilde{D}^I + C^{IJ}\psi_{H^1}^2\tilde{Q}_1^J\psi_D^I + \mathrm{h.c.}\Big)\\ &+\frac{u^I}{2}\Big(C^{JI*}\psi_{H^2}^1\psi_{Q^2}^J\tilde{U}^I + C^{JI*}\psi_{H^2}^1\psi_U^I\tilde{Q}_2^J\\ &+\mathrm{h.c.}\Big). \end{aligned} \tag{63}$$

Then we convert the two-component spinors into four-component spinors as discussed above:

$$\begin{aligned}\mathcal{L}_{\tilde{Q}q\kappa^{\pm}} &= C^{IJ}\bar{\kappa}_j^{+}\Bigg[(-gZ_{D_I}^{i,1}Z_{-}^{j,1} + \frac{d^I}{2}Z_{D_I}^{i,2}Z_{-}^{j,2})P_L\\ &+\frac{u^J}{2}Z_{+}^{j,2*}Z_{D_I}^{i,1}P_R\Bigg]\psi_{u^I}\tilde{D}_{I,i}^{+}\\ &+C^{IJ*}\bar{\kappa}_j^{-}\Bigg[\Big(-gZ_{U_J}^{i,1}Z_{+}^{j,1} + \frac{u^J}{2}Z_{U_J}^{i,2}Z_{+}^{j,2}\Big)P_L\\ &-\frac{d^I}{2}Z_{U_J}^{j,1*}Z_{-}^{j,2*}P_R\Bigg]\psi_{d^J}\tilde{U}_{Ji}^{-} + \mathrm{h.c.} \end{aligned} \tag{64}$$

Now ψ_{u^I}, ψ_{d^I} are four-component quark spinors of the Ith generation. The $\kappa_j^{-} = C\bar{\kappa}_j^{+T}$ (C is the charge-conjugation matrix) is a charged-conjugate state of κ_j^{+}, and κ_j^{+} is defined in (44). The Feynman rules are summarized in Fig. 9.

For the $\tilde{Q}q\kappa_i^0$ interactions we can write the parts in two-component notations as

$$\begin{aligned}\mathcal{L}_{\tilde{Q}q\kappa_i^0} &= \mathrm{i}\sqrt{2}\tilde{Q}^{I*}\Big(g\frac{\tau^3}{2}\lambda_A^3 + \frac{1}{6}g'\lambda_B\Big)\psi_Q^I\\ &-\mathrm{i}\sqrt{2}\tilde{Q}^{I}\Big(g\frac{\tau^3}{2}\bar{\lambda}_A^3 + \frac{1}{6}g'\bar{\lambda}_B\Big)\bar{\psi}_Q^I\\ &-\mathrm{i}\frac{2\sqrt{2}}{3}g'\tilde{U}^{I*}\lambda_B\psi_U^I\\ &+\mathrm{i}\frac{2\sqrt{2}}{3}g'\tilde{U}^{I}\bar{\lambda}_B\bar{\psi}_U^I + \mathrm{i}\frac{\sqrt{2}}{3}g'\tilde{D}^{I*}\lambda_B\psi_D^I\\ &-\mathrm{i}\frac{\sqrt{2}}{3}g'\tilde{D}^{I}\bar{\lambda}_B\bar{\psi}_D^I + \frac{d^I}{2}\Bigg[\psi_{H^1}^1\psi_{Q^2}^I\tilde{D}^I\\ &+\psi_{H^1}^1\psi_D^I\tilde{Q}_2^I + \mathrm{h.c.}\Bigg]\\ &-\frac{u^I}{2}\Bigg[\psi_{H^2}^2\psi_{Q^1}^I\tilde{U}^I + \psi_{H^2}^2\psi_U^I\tilde{Q}_1^I + \mathrm{h.c.}\Bigg]. \end{aligned} \tag{65}$$

After (65) is converted into four-component notations straightforwardly and when we use the definition for neutralino mass eigenstates, we arrive at

$$\begin{aligned}\mathcal{L}_{\tilde{Q}q\kappa_i^0} &= \kappa_j^0\Bigg\{\Bigg[\frac{e}{\sqrt{2}\sin\theta_{\mathrm{W}}\cos\theta_{\mathrm{W}}}Z_{U^I}^{i,1*}\\ &\times\Big(\cos\theta_{\mathrm{W}}Z_N^{i,2} + \frac{1}{3}\sin\theta_{\mathrm{W}}Z_N^{j,1}\Big)\\ &-\frac{u^I}{2}Z_{U^I}^{i,1*}Z_N^{j,4*}\Bigg]P_L\\ &+\Bigg[\frac{2\sqrt{2}}{3}g'Z_{U^I}^{i,2*}Z_N^{j,1} - \frac{u^I}{2}Z_{U^I}^{i,1*}Z_N^{j,4*}\Bigg]P_R\Bigg\}\psi_{u^I}\tilde{U}_{I,i}^{-}\\ &+\bar{\kappa}_j^0\Bigg\{\Bigg[\frac{e}{\sqrt{2}\sin\theta_{\mathrm{W}}\cos\theta_{\mathrm{W}}}Z_{D^I}^{i,1}\\ &\times\Big(-\cos\theta_{\mathrm{W}}Z_N^{i,2} + \frac{1}{3}\sin\theta_{\mathrm{W}}Z_N^{j,1}\Big)\\ &+\frac{d^I}{2}Z_{D^I}^{i,2}Z_N^{j,3}\Bigg]P_L + \Bigg[-\frac{\sqrt{2}}{3}g'Z_{D^I}^{i,2}Z_N^{j,1}\\ &+\frac{d^I}{2}Z_{D^I}^{i,1*}Z_N^{j,3*}\Bigg]P_R\Bigg\}\psi_{d^I}\tilde{D}_{I,i}^{+} + \mathrm{h.c.} \end{aligned} \tag{66}$$

Thus the Feynman rules for the interactions concerned may be depicted exactly as the last two diagrams in Fig. 9.

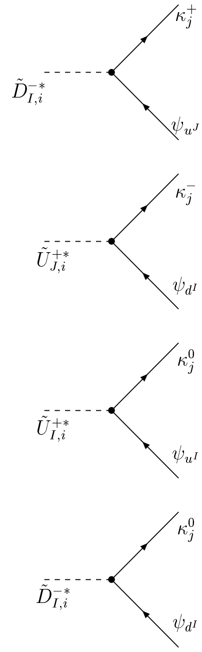
4 Numerical results

In this section we will analyze the mass spectrum of neutral Higgs, neutralinos and charginos numerically. We have obtained the mass matrices by taking the three types of sneutrinos with nonzero vacuum and $\epsilon_i \neq 0$ $(i = 1, 2, 3)$. However, the matrices are too big to arrive at the typical features. From now on we will assume that $\epsilon_1 = \epsilon_2 = 0$ and $\upsilon_{\tilde{\nu}_e} = \upsilon_{\tilde{\nu}_\mu} = 0$, i.e. only the τ-lepton number is violated. We have two reasons to make the assumption:

– Under the assumption we believe that the key features will not be lost but the mass matrices will turn out to be very simple.
– According to experimental indications, the τ-neutrino may be the heaviest among the three type neutrinos.

In the numerical calculations below, the input parameters are chosen as: $\alpha = e^2/4\pi = 1/128$, $m_Z = 91.19\,\mathrm{GeV}$, $m_W = 80.23\,\mathrm{GeV}$, and $m_\tau = 1.77\,\mathrm{GeV}$. For the unknown parameters m_1 and m_2 we assume $m_1 = m_2 = 1000\,\mathrm{GeV}$, and the upper limit on the τ-neutrino mass $m_{\nu_\tau} \leq 20\,\mathrm{MeV}$ is also seriously taken into account. Now let us first consider the mass matrix of neutralinos. When $\epsilon_1 = \epsilon_2 = 0$ and $\upsilon_{\tilde{\nu}_e} = \upsilon_{\tilde{\nu}_\mu} = 0$, (34) is simplified to

$$\mathcal{M}_N = \begin{pmatrix} 2m_1 & 0 & -\frac{1}{2}g'\upsilon_1 & \frac{1}{2}g'\upsilon_2 & -\frac{1}{2}g'\upsilon_{\tilde{\nu}_\tau}\\ 0 & 2m_2 & \frac{1}{2}g\upsilon_1 & -\frac{1}{2}g\upsilon_2 & \frac{1}{2}g\upsilon_{\tilde{\nu}_\tau}\\ -\frac{1}{2}g'\upsilon_1 & \frac{1}{2}g\upsilon_1 & 0 & -\frac{1}{2}\mu & 0\\ \frac{1}{2}g'\upsilon_2 & -\frac{1}{2}g\upsilon_2 & -\frac{1}{2}\mu & 0 & \frac{1}{2}\epsilon_3\\ -\frac{1}{2}g'\upsilon_{\tilde{\nu}_\tau} & \frac{1}{2}g\upsilon_{\tilde{\nu}_\tau} & 0 & \frac{1}{2}\epsilon_3 & 0 \end{pmatrix}. \tag{67}$$



$$iC^{IJ}[(-gZ_{D^I}^{i,1}Z_-^{j,1} + \tfrac{d^I}{2}Z_{D^I}^{i,2}Z_-^{j,2})P_L + \tfrac{u^J}{2}Z_+^{*j,2}Z_{D^I}^{i,1}P_R]$$

$$iC^{IJ*}[(-gZ_{U^J}^{i,1}Z_+^{j,1} + \tfrac{u^J}{2}Z_{U^J}^{i,2}Z_+^{j,2})P_L - \tfrac{d^I}{2}Z_-^{*j,2}Z_{U^J}^{*i,1}P_R]$$

$$i\{[\tfrac{\sqrt{g^2+g\prime^2}}{\sqrt{2}}Z_{U^I}^{i,1*}(\cos\theta_W Z_N^{i,2} + \tfrac{1}{3}\sin\theta_W Z_N^{j,1}) - \tfrac{u^I}{2}Z_{U^I}^{i,1*}Z_N^{j,4}]P_L$$
$$+[\tfrac{2\sqrt{2}}{3}g\prime Z_{U^I}^{i,2*}Z_N^{j,1} - \tfrac{u^I}{2}Z_{U^I}^{i,1*}Z_N^{j,4*}]P_R\}$$

$$i\{[\tfrac{\sqrt{g^2+g\prime^2}}{\sqrt{2}}Z_{D^I}^{i,1*}(-\cos\theta_W Z_N^{i,2} + \tfrac{1}{3}\sin\theta_W Z_N^{j,1}) + \tfrac{d^I}{2}Z_{D^I}^{i,1*}Z_N^{j,3}]P_L$$
$$+[-\tfrac{\sqrt{2}}{3}g\prime Z_{D^I}^{i,2*}Z_N^{j,1} + \tfrac{d^I}{2}Z_{D^I}^{i,1*}Z_N^{j,3*}]P_R\}$$

Fig. 9. Feynman rules for the coupling of quarks and squarks with charginos or neutralinos

As stated above, a strong restriction imposed on the matrix comes from $m_{\nu_\tau} \le 20\,\mathrm{MeV}$. In [21] the impact of this limit impacting on the parameter space is discussed; the numerical result indicates that a large value of ϵ_3 cannot be ruled out. In order to show the problem precisely, let us consider the equation for the eigenvalues of (67):

$$\begin{aligned}
\mathrm{Det}(\lambda - \mathcal{M}_N) &= \lambda^5 - 2(m_1+m_2)\lambda^4 \\
&+\Big(4m_1m_2 - \frac{1}{4}(\epsilon_3^2+\mu^2) - M_Z^2\Big)\lambda^3 \\
&+\bigg[\frac{1}{2}(m_1+m_2)(\epsilon_3^2+\mu^2) + 2(m_1+m_2)M_Z^2 \\
&+\frac{1}{4}(g^2+g'^2)\upsilon_2(-\mu\upsilon_1+\epsilon_3\upsilon_{\tilde{\nu}_\tau})\bigg]\lambda^2 \\
&+\bigg[-m_1m_2(\mu^2+\epsilon_3^2) + \frac{1}{16}(g^2+g'^2)(\epsilon_3\upsilon_1+\mu\upsilon_{\tilde{\nu}_\tau})^2 \\
&+\frac{1}{2}(g^2m_1+g'^2m_2)(\mu\upsilon_1\upsilon_2 - \epsilon_3\upsilon_2\upsilon_{\tilde{\nu}_\tau})\bigg]\lambda \\
&-\frac{1}{8}(g^2m_1+g'^2m_2)\Big(\mu\upsilon_{\tilde{\nu}_\tau}+\epsilon_3\upsilon_1\Big)^2 \\
&= \lambda^5 + \mathcal{A}_N\lambda^4 + \mathcal{B}_N\lambda^3 + \mathcal{C}_N\lambda^2 + \mathcal{D}_N\lambda + \mathcal{E}_N. \qquad (68)
\end{aligned}$$

For further discussions let us introduce new symbols X, Y:

$$\begin{aligned}
X &= \epsilon_3\cos\theta_\upsilon + \mu\sin\theta_\upsilon, \\
Y &= -\epsilon_3\sin\theta_\upsilon + \mu\cos\theta_\upsilon. \qquad (69)
\end{aligned}$$

Thus we have the coefficients of (68):

$$\begin{aligned}
\mathcal{A}_N &= -2(m_1+m_2), \\
\mathcal{B}_N &= -\frac{1}{4}(X^2+Y^2) + 4m_1m_2 - M_Z^2, \\
\mathcal{C}_N &= \frac{1}{2}(m_1+m_2)(X^2+Y^2) \\
&\quad +2(m_1+m_2)M_Z^2 - M_Z^2\cos\beta\sin\beta Y, \\
\mathcal{D}_N &= -m_1m_2(X^2+Y^2) + \frac{1}{4}M_Z^2\cos^2\beta X^2 \\
&\quad +\frac{1}{2}(g^2m_1+g'^2)\upsilon^2\sin\beta\cos\beta Y, \\
\mathcal{E}_N &= -\frac{1}{8}(g^2m_1+g'^2m_2)\upsilon^2\cos^2\beta X^2. \qquad (70)
\end{aligned}$$

If we fix the τ-neutrino mass m_{ν_τ} as an input parameter, then the equation $\mathrm{Det}(\lambda - \mathcal{M}_N) = 0$ can be written as

$$\Big(\lambda - m_{\nu_e}\Big)\Big(\lambda^4 + \mathcal{A}'_N\lambda^3 + \mathcal{B}'_N\lambda^2 + \mathcal{C}'_N\lambda + \mathcal{D}'_N\Big) = 0. \qquad (71)$$

The coefficients $\mathcal{A}'_N, \mathcal{B}'_N, \mathcal{C}'_N, \mathcal{D}'_N$ are related to the "original" $\mathcal{A}_N, \mathcal{B}_N, \mathcal{C}_N, \mathcal{D}_N$ and $\mathcal{E}_N$ as follows:

$$\begin{aligned}
\mathcal{A}'_N &= \mathcal{A}_N + m_{\nu_\tau}, \\
\mathcal{B}'_N &= \mathcal{B}_N + m_{\nu_\tau}\mathcal{A}'_N, \\
\mathcal{C}'_N &= \mathcal{C}_N + m_{\nu_\tau}\mathcal{B}'_N, \\
\mathcal{D}'_N &= \mathcal{D}_N + m_{\nu_\tau}\mathcal{C}'_N = -\frac{\mathcal{E}_N}{m_{\nu_\tau}}. \qquad (72)
\end{aligned}$$

In order to obtain the masses of the other four neutralinos, let us solve (71) by a numerical method. In Fig. 10 we plot the mass of the lightest neutralino versus X. The three lines correspond to $m_{\nu_\tau} = 20\,\mathrm{MeV}$, $2\,\mathrm{MeV}$ and $0.2\,\mathrm{MeV}$ respectively. From the figure we find that the curve corresponding to $m_{\nu_\tau} = 20\,\mathrm{MeV}$ is the lowest, and the next to lowest corresponds to $m_{\nu_\tau} = 2\,\mathrm{MeV}$, so the tendency is that the curves are going "up" as the τ-neutrino mass is decreasing. If the mass of the lightest neutralino is not too large (e.g. $m_{\kappa_0^1} \le 300\,\mathrm{GeV}$), the absolute value of X cannot be very large (e.g. $|X| \le 800\,\mathrm{GeV}$).

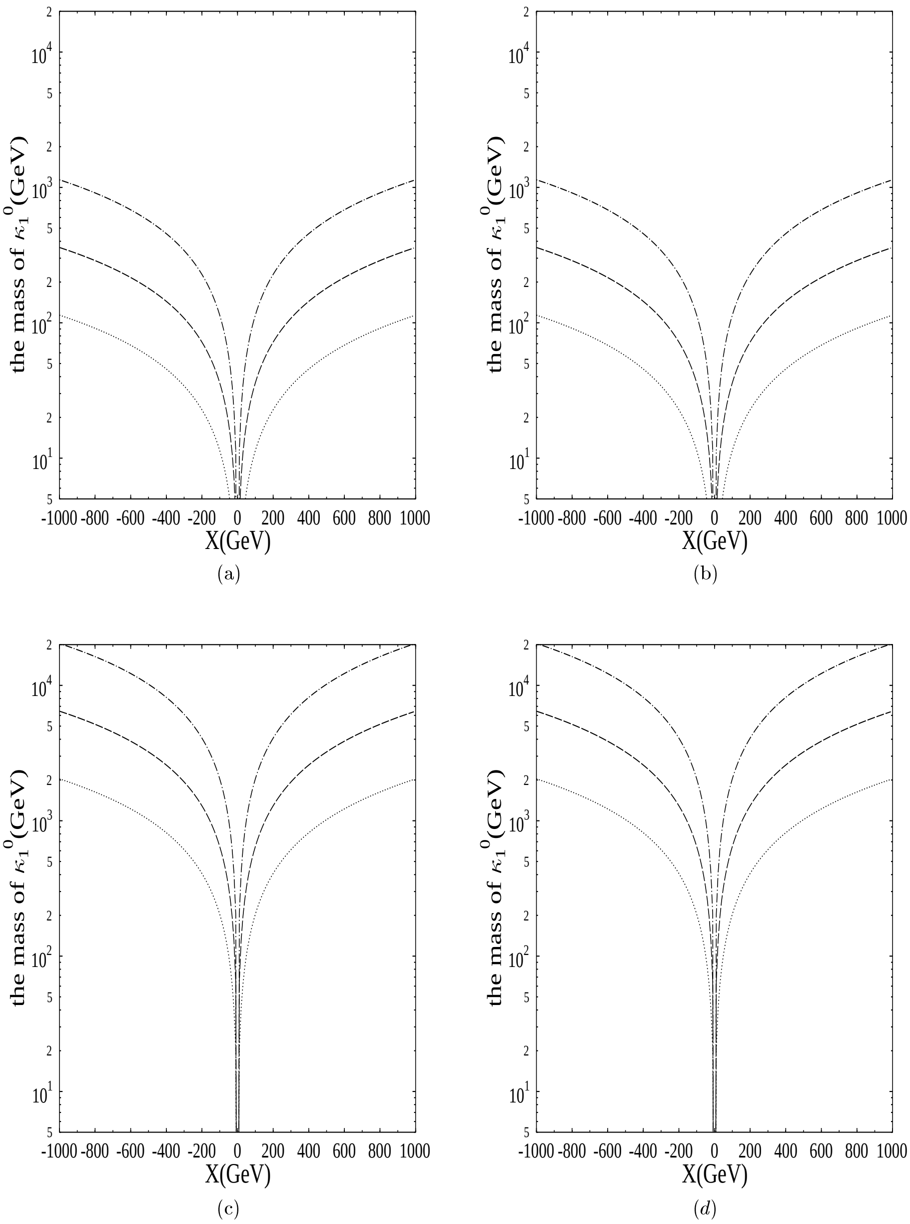


Fig. 10a–d. The mass of the lightest neutralino as a function of X. The parameters are assigned to be $m_1 = m_2 = 3000\,\mathrm{GeV}$ and **a** $\tan\beta = 20$, $\tan\theta_\upsilon = 20$; **b** $\tan\beta = 20$, $\tan\theta_\upsilon = 0.5$; **c** $\tan\beta = 0.5$, $\tan\theta_\upsilon = 20$; **d** $\tan\beta = 0.5$, $\tan\theta_\upsilon = 0.5$. The dotted–dashed lines correspond to $m_{\nu_\tau} = 0.2\,\mathrm{MeV}$, the dashed lines correspond to $m_{\nu_\tau} = 2\,\mathrm{MeV}$, and the dotted lines correspond to $m_{\nu_\tau} = 20\,\mathrm{MeV}$

As for the mass of the charginos, when $\epsilon_1 = \epsilon_2 = 0$, and $\upsilon_{\tilde{\nu}_e} = \upsilon_{\tilde{\nu}_\mu} = 0$, (41) becomes

$$\mathcal{M}_\mathrm{C} = \begin{pmatrix} 2m_2 & \frac{e\upsilon_2}{\sqrt{2}S_\mathrm{W}} & 0 \\ \frac{e\upsilon_1}{\sqrt{2}S_\mathrm{W}} & \mu & \frac{l_3\upsilon_{\tilde{\nu}_\tau}}{\sqrt{2}} \\ \frac{e\upsilon_{\tilde{\nu}_\tau}}{\sqrt{2}S_\mathrm{W}} & \epsilon_3 & \frac{l_3\upsilon_1}{\sqrt{2}} \end{pmatrix}, \tag{73}$$

Because m_τ^2 should be the lightest eigenvalue of the matrix $\mathcal{M}_\mathrm{C}^\dagger\mathcal{M}_\mathrm{C}$, after the eigenvalue m_τ^2 is taken away, the surviving eigenvalue equation becomes

$$\lambda^2 - \mathcal{A}_\mathrm{C}\lambda + \mathcal{B}_\mathrm{C} = 0. \tag{74}$$

Here

$$\mathcal{A}_\mathrm{C} = X^2 + Y^2 + 4m_2^2 + l_3^2\frac{\upsilon_1^2 + \upsilon_{\tilde{\nu}_\tau}^2}{2} + \frac{e^2\upsilon^2}{2S_\mathrm{W}^2},$$

$$\mathcal{B}_\mathrm{C} = \frac{2l_3^2}{m_\tau^2}\Bigg\{m_2\upsilon\cos\beta Y + \frac{e^2}{4S_\mathrm{W}^2}\upsilon^3 \times \cos^2\beta\sin\beta\Big(\sin^2\theta_\upsilon - \cos^2\theta_\upsilon\Big)\Bigg\}^2, \tag{75}$$

where the parameters X, Y are defined by (69). Therefore the masses of the other two charginos are expressed as

$$m^2_{\kappa^\pm_{1,2}} = \frac{1}{2}\Big\{\mathcal{A}_\mathrm{C} \mp \sqrt{\mathcal{A}_\mathrm{C}^2 - 4\mathcal{B}_\mathrm{C}}\Big\}. \tag{76}$$

The parameter l_3 can be fixed by the condition $\mathrm{Det}|m_\tau^2 - \mathcal{M}_\mathrm{C}^\dagger\mathcal{M}_\mathrm{C}| = 0$. In Fig. 11 we plot the mass of the lightest chargino versus X. The three lines correspond to $m_{\nu_\tau} = 20\,\mathrm{MeV}$, $2\,\mathrm{MeV}$ and $0.2\,\mathrm{MeV}$ respectively. As we found in

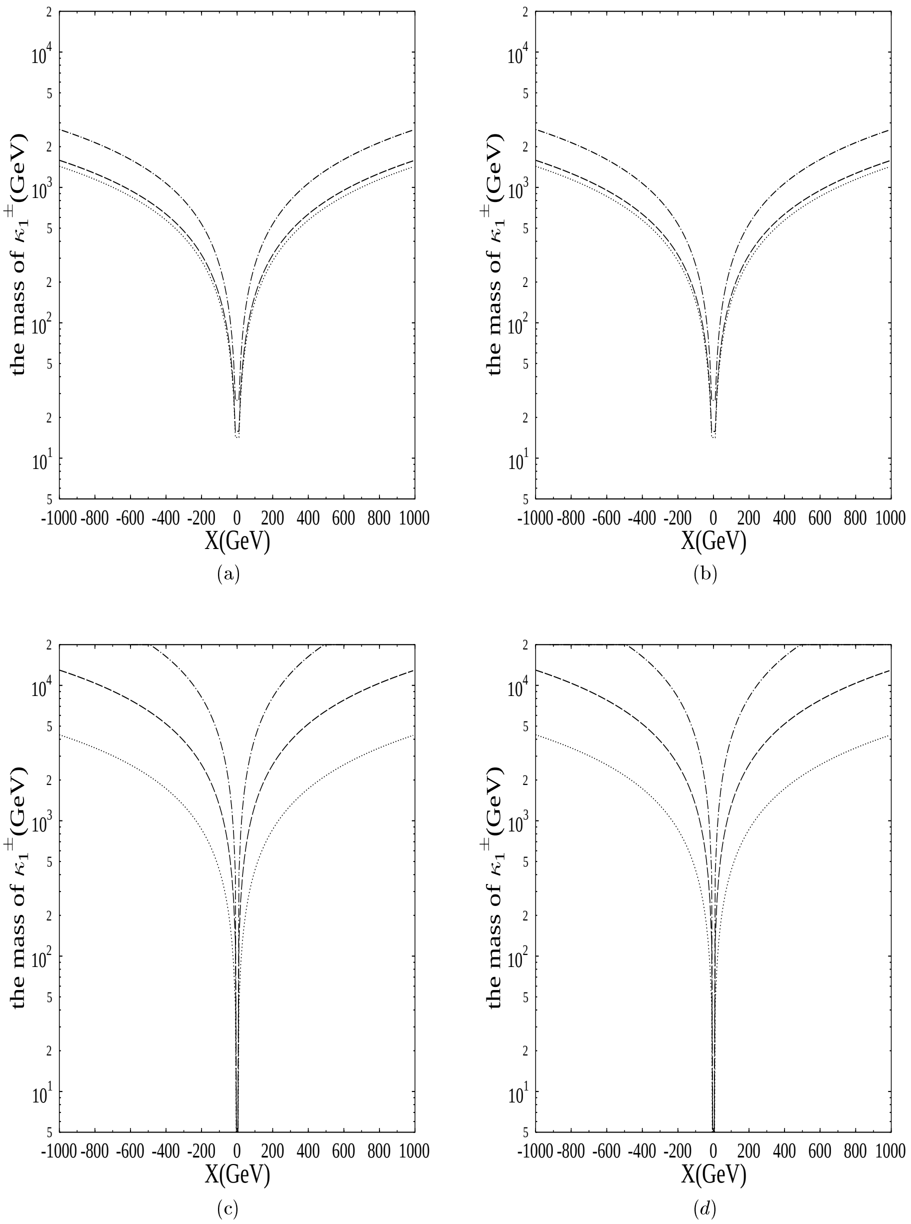


Fig. 11a–d. The mass of the lightest chargino as a function of X. The parameters are assigned to be $m_1 = m_2 = 3000\,\text{GeV}$ and **a** $\tan\beta = 20$, $\tan\theta_\upsilon = 20$; **b** $\tan\beta = 20$, $\tan\theta_\upsilon = 0.5$; **c** $\tan\beta = 0.5$, $\tan\theta_\upsilon = 20$; **d** $\tan\beta = 0.5$, $\tan\theta_\upsilon = 0.5$. The dotted–dashed lines correspond to $m_{\nu_\tau} = 0.2\,\text{MeV}$, the dashed lines correspond to $m_{\nu_\tau} = 2\,\text{MeV}$, and the dotted lines correspond to $m_{\nu_\tau} = 20\,\text{MeV}$

the case of the neutralinos, we find that the curve corresponding to $m_{\nu_\tau} = 20\,\text{MeV}$ is the lowest, and the next to lowest corresponds to $m_{\nu_\tau} = 2\,\text{MeV}$; the tendency is very similar to the case for neutralinos. This can be understood as follows: when the values of m_1, m_2, $\tan\beta$, $\tan\theta_\upsilon$ and X are fixed, the value of Y will be fixed by the mass of the τ-neutrino. In the numerical computation we find that the absolute value of Y becomes small as the m_{ν_τ} changes large. This is the reason why the curve corresponding to $m_{\nu_\tau} = 20\,\text{MeV}$ is the lowest among the three curves which we have computed here.

Now we turn to a discussion of the mass matrix of the neutral Higgs. Under the same assumption the mass matrix for CP-even Higgs reduces to

$$\mathcal{M}^2_{\text{even}} = \tag{77}$$

$$\begin{pmatrix} r_{11} & -\frac{g^2+g'^2}{4}\upsilon_1\upsilon_2 - B\mu & \frac{g^2+g'^2}{4}\upsilon_1\upsilon_{\tilde{\nu}_\tau} - \mu\epsilon_3 \\ -\frac{g^2+g'^2}{4}\upsilon_1\upsilon_2 - B\mu & r_{22} & -\frac{g^2+g'^2}{4}\upsilon_2\upsilon_{\tilde{\nu}_\tau} + B_3\epsilon_3 \\ \frac{g^2+g'^2}{4}\upsilon_1\upsilon_{\tilde{\nu}_\tau} - \mu\epsilon_3 & -\frac{g^2+g'^2}{4}\upsilon_2\upsilon_{\tilde{\nu}_\tau} + B_3\epsilon_3 & r_{33} \end{pmatrix},$$

with

$$r_{11} = \frac{g^2+g'^2}{8}(3\upsilon_1^2 - \upsilon_2^2 + \upsilon_{\tilde{\nu}_\tau}^2) + |\mu|^2 + m^2_{H^1}$$
$$= \frac{g^2+g'^2}{4}\upsilon_1^2 + \mu\epsilon_3\frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_1} + B\mu\frac{\upsilon_2}{\upsilon_1},$$

$$r_{22} = \frac{g^2+g'^2}{8}(-\upsilon_1^2 + 3\upsilon_2^2 - \upsilon_{\tilde{\nu}_\tau}^2) + |\mu|^2 + |\epsilon_3|^2 + m^2_{H^2}$$
$$= \frac{g^2+g'^2}{4}\upsilon_2^2 + B\mu\frac{\upsilon_1}{\upsilon_2} - B_3\epsilon_3\frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_2},$$

$$r_{33} = \frac{g^2+g'^2}{8}(\upsilon_1^2 - \upsilon_2^2 + 3\upsilon_{\tilde{\nu}_\tau}^2) + |\epsilon_3|^2 + m^2_{L^3}$$

$$= \frac{g^2 + g'^2}{4} \upsilon_{\tilde{\nu}_\tau}^2 + \mu\epsilon_3 \frac{\upsilon_1}{\upsilon_{\tilde{\nu}_\tau}} - B_3\epsilon_3 \frac{\upsilon_2}{\upsilon_{\tilde{\nu}_\tau}}. \quad (78)$$

The mass matrix of CP-odd Higgs reduces to

$$\mathcal{M}_{\text{odd}}^2 = \begin{pmatrix} s_{11} & B\mu & -\mu\epsilon_3 \\ B\mu & s_{22} & -B_3\epsilon_3 \\ -\mu\epsilon_3 & -B_3\epsilon_3 & s_{33} \end{pmatrix}, \quad (79)$$

with

$$\begin{aligned} s_{11} &= \frac{g^2 + g'^2}{8}(\upsilon_1^2 - \upsilon_2^2 + \upsilon_{\tilde{\nu}_\tau}^2) + |\mu|^2 + m_{H^1}^2 \\ &= \mu\epsilon_3 \frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_1} + B\mu \frac{\upsilon_2}{\upsilon_1}, \\ s_{22} &= -\frac{g^2 + g'^2}{8}(\upsilon_1^2 - \upsilon_2^2 + \upsilon_{\tilde{\nu}_\tau}^2) + |\mu|^2 + |\epsilon_3|^2 + m_{H^2}^2 \\ &= B\mu \frac{\upsilon_1}{\upsilon_2} - B_3\epsilon_3 \frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_2}, \\ s_{33} &= \frac{g^2 + g'^2}{8}(\upsilon_1^2 - \upsilon_2^2 + \upsilon_{\tilde{\nu}_\tau}^2) + |\epsilon_3|^2 + m_{L^3}^2 \\ &= \mu\epsilon_3 \frac{\upsilon_1}{\upsilon_{\tilde{\nu}_\tau}} - B_3\epsilon_3 \frac{\upsilon_2}{\upsilon_{\tilde{\nu}_\tau}}. \end{aligned} \quad (80)$$

We introduce the following variables:

$$\begin{aligned} X_{\text{s}} &= B\mu, \\ Y_{\text{s}} &= \mu\epsilon_3, \\ Z_{\text{s}} &= B_3\epsilon_3. \end{aligned} \quad (81)$$

The masses of the neutral Higgs can be determined from $X_{\text{s}}, Y_{\text{s}}, Z_{\text{s}}$ and $\tan\beta$, $\tan\theta_\upsilon$. For the masses of CP-odd Higgs we define

$$\begin{aligned} \mathcal{A} &= X_{\text{s}}\left(\frac{\upsilon_1}{\upsilon_2} + \frac{\upsilon_2}{\upsilon_1}\right) + Y_{\text{s}}\left(\frac{\upsilon_1}{\upsilon_{\tilde{\nu}_\tau}} + \frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_1}\right) \\ &\quad - Z_{\text{s}}\left(\frac{\upsilon_2}{\upsilon_{\tilde{\nu}_\tau}} + \frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_2}\right), \\ \mathcal{B} &= -Y_{\text{s}}Z_{\text{s}}\left(\frac{\upsilon_1}{\upsilon_2} + \frac{\upsilon_2}{\upsilon_1}\right) - X_{\text{s}}Z_{\text{s}}\left(\frac{\upsilon_1}{\upsilon_{\tilde{\nu}_\tau}} + \frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_1}\right) \\ &\quad + X_{\text{s}}Y_{\text{s}}\left(\frac{\upsilon_2}{\upsilon_{\tilde{\nu}_\tau}} + \frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_2}\right) + X_{\text{s}}Y_{\text{s}}\frac{\upsilon_1^2}{\upsilon_2\upsilon_{\tilde{\nu}_\tau}} \\ &\quad - X_{\text{s}}Z_{\text{s}}\frac{\upsilon_2^2}{\upsilon_1\upsilon_{\tilde{\nu}_\tau}} - Y_{\text{s}}Z_{\text{s}}\frac{\upsilon_{\tilde{\nu}_\tau}^2}{\upsilon_1\upsilon_2}. \end{aligned} \quad (82)$$

The masses of the two CP-odd Higgs can be written as

$$m_{H_{3+2,3}^0}^2 = \frac{1}{2}\left(\mathcal{A} \mp \sqrt{\mathcal{A}^2 - 4\mathcal{B}}\right). \quad (83)$$

In Fig. 12 we plot the mass of the lightest CP-odd Higgs versus the mass of the lightest CP-even Higgs, where the ranges of the parameters are: $-10^5\,\text{GeV}^2 \le X_{\text{s}}, Y_{\text{s}}, Z_{\text{s}} \le 10^5\,\text{GeV}^2$ and $0.5 \le \tan\beta, \tan\theta_\upsilon \le 50$. From Fig. 12 we can find that there are no limits on $m_{H_5^0}$ when we change those parameters in the above ranges. As for the lightest CP-even Higgs, the difference from the MSSM with R-parity is that $m_{H_1^0}$ can be larger than m_Z at the tree level. This can be understood from (26). Under the assumptions we have

$$m_{H_1^0}^2 \le m_{H_3^0} m_Z \cos 2\beta \frac{1 - \frac{1}{2}\frac{m_Z^2}{m_{H_3^0}^2}}{1 - \frac{1}{2}\left(\frac{m_Z^2\cos^2 2\beta}{m_{H_3^0}^2}\right)^{\frac{1}{2}}}, \quad (84)$$

where $m_{H_3^0}$ is the mass of the heaviest CP-even Higgs. We cannot give a stringent limit on it, as we can in the MSSM with R-parity.

In summary, we have analyzed the mass spectrum in the MSSM with bilinear R-parity violation. From the restriction $m_{\nu_\tau} \le 20\,\text{MeV}$ we cannot rule out the possibilities with large ϵ_3 and $\upsilon_{\tilde{\nu}_\tau}$. We also derived the Feynman rules in the 't Hooft–Feynman gauge, which are convenient when we study the phenomenology beyond the tree level in the model. Recent experimental signals of neutrino masses and mixing may provide the first glimpses of lepton number violation effects. In [22] the experimental constraints of neutrino oscillations on the parameter space of the model are discussed. There both the fermionic and scalar sectors are considered, and it is found that a large area of the parameter space is allowed. Here we would also like to point out that in some references the $0\nu\beta\beta$-decay is analyzed in the model [23], and new stringent upper limits on the first generation R-parity violating parameters, ϵ_1 and $\upsilon_{\tilde{\nu}_e}$, are obtained; for the other two generations there are no very serious restrictions on the upper limits of the R-parity violating parameters. As for other interesting processes in the model, they are discussed in [24].

Acknowledgements. This work was supported in part by the National Natural Science Foundation of China and Grant No. LWLZ-1298 of the Chinese Academy of Sciences.

Appendix A: The mass matrix of charged Higgs

In the case of charged Higgs, with the current basis $\Phi_{\text{c}} = (H_2^{1*}, H_1^2, \tilde{L}_2^{1*}, \tilde{L}_2^{2*}, \tilde{L}_2^{3*}, \tilde{R}^1, \tilde{R}^2, \tilde{R}^3)$, the symmetric matrix $\mathcal{M}_{\text{c}}^2$ is given as

$$\begin{aligned} \mathcal{M}_{\text{c}1,1}^2 &= \frac{g^2}{4}\upsilon_1^2 - \frac{g^2 - g'^2}{8}(\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon_{\tilde{\nu}_I}^2) \\ &\quad + \mu^2 + \sum_I \frac{1}{2} l_I^2 \upsilon_{\tilde{\nu}_I}^2 + m_{H^1}^2 \\ &= \frac{g^2}{4}(\upsilon_2^2 - \sum_I \upsilon_{\tilde{\nu}_I}^2) \\ &\quad + \sum_I \frac{1}{2} l_I^2 \upsilon_{\tilde{\nu}_I}^2 + \sum_I \mu\epsilon_I \frac{\upsilon_{\tilde{\nu}_I}}{\upsilon_1} + B\mu\frac{\upsilon_2}{\upsilon_1}, \\ \mathcal{M}_{\text{c}1,2}^2 &= \frac{g^2}{4}\upsilon_1\upsilon_2 + B\mu, \end{aligned}$$

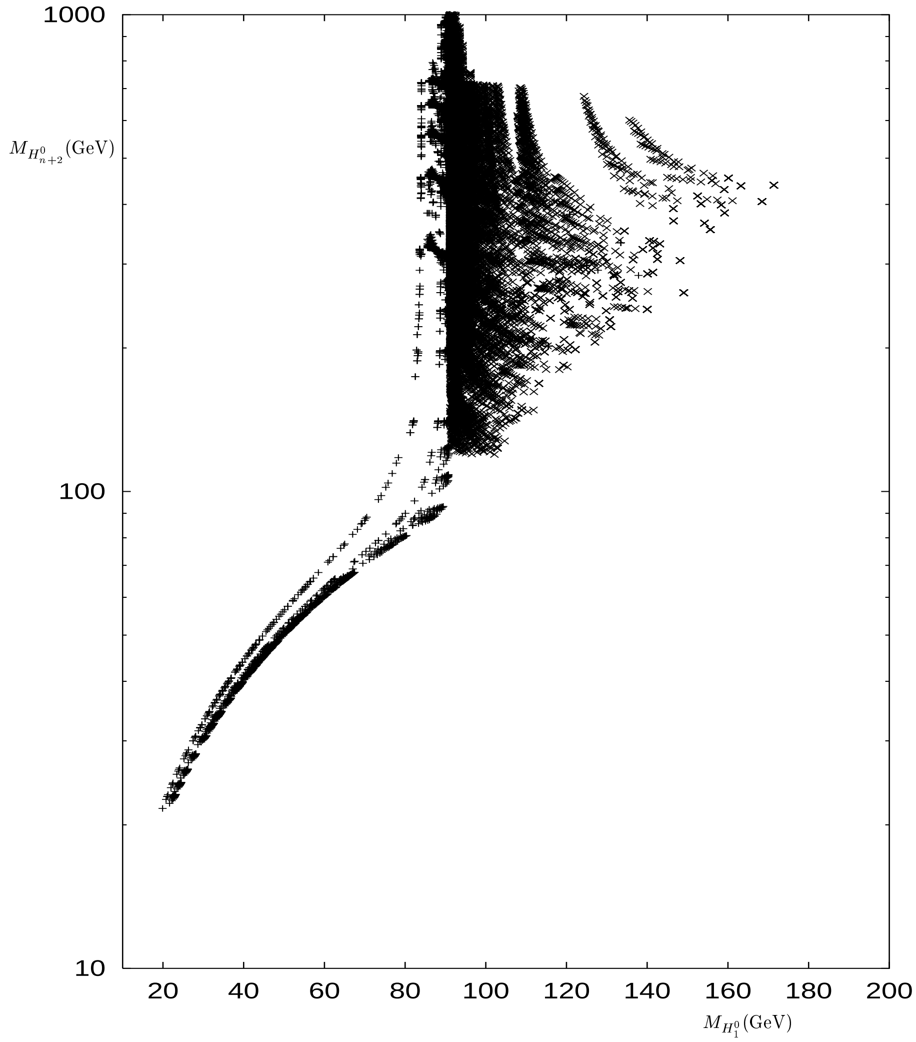


Fig. 12. The mass of the lightest CP-odd Higgs as a function of the mass of the lightest CP-even Higgs ($n = 3$). The range of parameters are assigned to be $-10^5\,\text{GeV}^2 \le X_\text{s}, Y_\text{s}, Z_\text{s} \le 10^5\,\text{GeV}^2$ and $0.5 \le \tan\beta \le 50$, $0.5 \le \tan\theta_\upsilon \le 50$

$$\mathcal{M}^2_{\text{c}1,3} = \frac{g^2}{4}\upsilon_1\upsilon_{\tilde\nu_e} - \mu\epsilon_1 - \frac{1}{2}l_1^2\upsilon_1\upsilon_{\tilde\nu_e},$$

$$\mathcal{M}^2_{\text{c}1,4} = \frac{g^2}{4}\upsilon_1\upsilon_{\tilde\nu_\mu} - \mu\epsilon_2 - \frac{1}{2}l_2^2\upsilon_1\upsilon_{\tilde\nu_\mu},$$

$$\mathcal{M}^2_{\text{c}1,5} = \frac{g^2}{4}\upsilon_1\upsilon_{\tilde\nu_\tau} - \mu\epsilon_3 - \frac{1}{2}l_3^2\upsilon_1\upsilon_{\tilde\nu_\tau},$$

$$\mathcal{M}^2_{\text{c}1,6} = \frac{1}{\sqrt{2}}l_1\epsilon_1\upsilon_2 + l_{s_1}\frac{\mu\upsilon_{\tilde\nu_e}}{\sqrt{2}},$$

$$\mathcal{M}^2_{\text{c}1,7} = \frac{1}{\sqrt{2}}l_2\epsilon_2\upsilon_2 + l_{s_2}\frac{\mu\upsilon_{\tilde\nu_\mu}}{\sqrt{2}},$$

$$\mathcal{M}^2_{\text{c}1,8} = \frac{1}{\sqrt{2}}l_3\epsilon_3\upsilon_2 + l_{s_3}\frac{\mu\upsilon_{\tilde\nu_\tau}}{\sqrt{2}},$$

$$\begin{aligned}\mathcal{M}^2_{\text{c}2,2} &= \frac{g^2}{4}\upsilon_2^2 + \frac{1}{8}(g^2 - g'^2)(\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon_{\tilde\nu_I}^2) \\ &\quad +\mu^2 + \sum_I \epsilon_I^2 + m^2_{H^2} \\ &= \frac{g^2}{4}(\upsilon_1^2 + \sum_I \upsilon_{\tilde\nu_I}^2) - \sum_I B_I\epsilon_I\frac{\upsilon_{\tilde\nu_I}}{\upsilon_2} + B\mu\frac{\upsilon_1}{\upsilon_2},\end{aligned}$$

$$\mathcal{M}^2_{\text{c}2,3} = \frac{g^2}{4}\upsilon_2\upsilon_{\tilde\nu_e} - B_1\epsilon_1,$$

$$\mathcal{M}^2_{\text{c}2,4} = \frac{g^2}{4}\upsilon_2\upsilon_{\tilde\nu_\mu} - B_2\epsilon_2,$$

$$\mathcal{M}^2_{\text{c}2,5} = \frac{g^2}{4}\upsilon_2\upsilon_{\tilde\nu_\tau} - B_3\epsilon_3,$$

$$\mathcal{M}^2_{\text{c}2,6} = \frac{l_1}{\sqrt{2}}\mu\upsilon_{\tilde\nu_e} + \frac{l_1}{\sqrt{2}}\epsilon_1\upsilon_1,$$

$$\mathcal{M}^2_{\text{c}2,7} = \frac{l_2}{\sqrt{2}}\mu\upsilon_{\tilde\nu_\mu} + \frac{l_2}{\sqrt{2}}\epsilon_2\upsilon_1,$$

$$\mathcal{M}^2_{\text{c}2,8} = \frac{l_3}{\sqrt{2}}\mu\upsilon_{\tilde\nu_\tau} + \frac{l_3}{\sqrt{2}}\epsilon_3\upsilon_1,$$

$$\begin{aligned}\mathcal{M}^2_{\text{c}3,3} &= \frac{g^2}{4}\upsilon_{\tilde\nu_e}^2 - \frac{1}{8}(g^2 - g'^2)(\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon_{\tilde\nu_I}^2) \\ &\quad +\epsilon_1^2 + \frac{l_1^2}{2}\upsilon_1^2 + m^2_{L^1} \\ &= \frac{g^2}{4}(\upsilon_2^2 - \upsilon_1^2) + \epsilon_1\frac{\mu\upsilon_1}{\upsilon_{\tilde\nu_e}} - B_1\frac{\epsilon_1\upsilon_2}{\upsilon_{\tilde\nu_e}} + \frac{l_1^2}{2}\upsilon_1^2\end{aligned}$$

$$
\begin{aligned}
&-\epsilon_1\epsilon_2\frac{\upsilon_{\tilde\nu_\mu}}{\upsilon_{\tilde\nu_e}}-\epsilon_1\epsilon_3\frac{\upsilon_{\tilde\nu_\tau}}{\upsilon_{\tilde\nu_e}}-\frac{g^2}{4}(\upsilon_{\tilde\nu_\mu}^2+\upsilon_{\tilde\nu_\tau}^2),\\
\mathcal{M}^2_{\mathrm{c}3,4}&=\frac{g^2}{4}\upsilon_{\tilde\nu_e}\upsilon_{\tilde\nu_\mu}+\epsilon_1\epsilon_2,\\
\mathcal{M}^2_{\mathrm{c}3,5}&=\frac{g^2}{4}\upsilon_{\tilde\nu_e}\upsilon_{\tilde\nu_\tau}+\epsilon_1\epsilon_3,\\
\mathcal{M}^2_{\mathrm{c}3,6}&=\frac{1}{\sqrt2}l_1\mu\upsilon_2+\frac{1}{\sqrt2}l_{s_1}\mu\upsilon_1,\\
\mathcal{M}^2_{\mathrm{c}3,7}&=0,\\
\mathcal{M}^2_{\mathrm{c}3,8}&=0,\\
\mathcal{M}^2_{\mathrm{c}4,4}&=\frac{g^2}{4}\upsilon_{\tilde\nu_\mu}^2-\frac18(g^2-g'^2)(\upsilon_1^2-\upsilon_2^2+\sum_I\upsilon_{\tilde\nu_I}^2)\\
&\quad+\epsilon_2^2+\frac{l_2^2}{2}\upsilon_1^2+m^2_{L^2}\\
&=\frac{g^2}{4}(\upsilon_2^2-\upsilon_1^2)+\epsilon_2\frac{\mu\upsilon_1}{\upsilon_{\tilde\nu_\mu}}-B_2\frac{\epsilon_2\upsilon_2}{\upsilon_{\tilde\nu_\mu}}+\frac{l_2^2}{2}\upsilon_1^2\\
&\quad-\epsilon_1\epsilon_2\frac{\upsilon_{\tilde\nu_e}}{\upsilon_{\tilde\nu_\mu}}-\epsilon_2\epsilon_3\frac{\upsilon_{\tilde\nu_\tau}}{\upsilon_{\tilde\nu_\mu}}-\frac{g^2}{4}(\upsilon_{\tilde\nu_e}^2+\upsilon_{\tilde\nu_\tau}^2),\\
\mathcal{M}^2_{\mathrm{c}4,5}&=\frac{g^2}{4}\upsilon_{\tilde\nu_\mu}\upsilon_{\tilde\nu_\tau},\\
\mathcal{M}^2_{\mathrm{c}4,6}&=0,\\
\mathcal{M}^2_{\mathrm{c}4,7}&=\frac{1}{\sqrt2}l_2\mu\upsilon_2-\frac{1}{\sqrt2}l_{s_2}\mu\upsilon_1,\\
\mathcal{M}^2_{\mathrm{c}4,8}&=0,\\
\mathcal{M}^2_{\mathrm{c}5,5}&=\frac{g^2}{4}\upsilon_{\tilde\nu_\tau}^2-\frac18(g^2-g'^2)(\upsilon_1^2-\upsilon_2^2+\sum_I\upsilon_{\tilde\nu_I}^2)\\
&\quad+\epsilon_3^2+\frac{l_3^2}{2}\upsilon_1^2+m^2_{L^3}\\
&=\frac{g^2}{4}(\upsilon_2^2-\upsilon_1^2)+\epsilon_3\frac{\mu\upsilon_1}{\upsilon_{\tilde\nu_\tau}}-B_3\frac{\epsilon_3\upsilon_2}{\upsilon_{\tilde\nu_\tau}}+\frac{l_3^2}{2}\upsilon_1^2\\
&\quad-\epsilon_1\epsilon_3\frac{\upsilon_{\tilde\nu_e}}{\upsilon_{\tilde\nu_\tau}}-\epsilon_2\epsilon_3\frac{\upsilon_{\tilde\nu_\mu}}{\upsilon_{\tilde\nu_\tau}}-\frac{g^2}{4}(\upsilon_{\tilde\nu_e}^2+\upsilon_{\tilde\nu_\mu}^2),\\
\mathcal{M}^2_{\mathrm{c}5,6}&=0,\\
\mathcal{M}^2_{\mathrm{c}5,7}&=0,\\
\mathcal{M}^2_{\mathrm{c}5,8}&=\frac{1}{\sqrt2}l_3\mu\upsilon_{\tilde\nu_\tau}-\frac{1}{\sqrt2}l_{s_3}\mu\upsilon_1,\\
\mathcal{M}^2_{\mathrm{c}6,6}&=-\frac{g'^2}{4}(\upsilon_1^2-\upsilon_2^2+\sum_I\upsilon_{\tilde\nu_I}^2)+\frac12 l_1^2(\upsilon_1^2+\upsilon_{\tilde\nu_e}^2)+m^2_{R^1},\\
\mathcal{M}^2_{\mathrm{c}6,7}&=\frac12 l_1l_2\upsilon_{\tilde\nu_e}\upsilon_{\tilde\nu_\mu},\\
\mathcal{M}^2_{\mathrm{c}6,8}&=\frac12 l_1l_3\upsilon_{\tilde\nu_e}\upsilon_{\tilde\nu_\tau},\\
\mathcal{M}^2_{\mathrm{c}7,7}&=-\frac{g'^2}{4}(\upsilon_1^2-\upsilon_2^2+\sum_I\upsilon_{\tilde\nu_I}^2)+\frac12 l_1^2(\upsilon_1^2+\upsilon_{\tilde\nu_\mu}^2)+m^2_{R^2},\\
\mathcal{M}^2_{\mathrm{c}7,8}&=\frac12 l_2l_3\upsilon_{\tilde\nu_\mu}\upsilon_{\tilde\nu_\tau},\\
\mathcal{M}^2_{\mathrm{c}8,8}&=-\frac{g'^2}{4}(\upsilon_1^2-\upsilon_2^2+\sum_I\upsilon_{\tilde\nu_I}^2)\\
&\quad+\frac12 l_3^2(\upsilon_1^2+\upsilon_{\tilde\nu_\tau}^2)+m^2_{R^3}.
\end{aligned}\tag{A1}
$$

Note that in order to obtain (A1), (12) is used sometimes.

Appendix B: The mixing of the squarks

In a general case the matrix of squark mixing should be 6×6. Under our assumptions we do not consider squark mixing between different generations. From superpotential (2) and the soft breaking terms we find that the up squark mass matrix of the Ith generation can be written as

$$
\mathcal{M}^2_{U^I}=\tag{B1}
$$
$$
\begin{pmatrix}
\frac{1}{24}(3g^2-g'^2)(\upsilon^2-2\upsilon_2^2)+\frac{u_I^2}{2}\upsilon_2^2+m^2_{Q^I} & \frac{1}{\sqrt2}(u_I\mu\upsilon_1-u_I\sum_{J=1}^3\epsilon_J\upsilon_{\tilde\nu_J}-u_{S_I}\mu\upsilon_2)\\
\frac{1}{\sqrt2}(u_I\mu\upsilon_1-u_I\sum_{J=1}^3\epsilon_J\upsilon_{\tilde\nu_J}-u_{S_I}\mu\upsilon_2) & \frac16 g'^2(\upsilon^2-2\upsilon_2^2)+\frac{u_I^2}{2}\upsilon_2^2+m^2_{U^I}
\end{pmatrix},
$$

where $I=(1,2,3)$ is the index of the generations. The current eigenstates $\tilde Q_1^I$ and $\tilde U^I$ connect to the two physical (mass) eigenstates $\tilde U_I^i(i=(1,\,2)$ through

$$
\tilde U_I^i=Z^{i,1}_{U^I}\tilde Q_1^I+Z^{i,2}_{U^I}\tilde U^I,\tag{B2}
$$

and Z_{U^I} is determined by the condition

$$
Z^\dagger_{U^I}\mathcal{M}^2_{U^I}Z_{U^I}=\mathrm{diag}(M^2_{U_I^1},M^2_{U_I^2}).\tag{B3}
$$

In a similar way we can give the down squark mass matrix of the Ith generation:

$$
\mathcal{M}^2_{D^I}=\tag{B4}
$$
$$
\begin{pmatrix}
-\frac{1}{24}(3g^2+g'^2)(\upsilon^2-2\upsilon_2^2)+\frac{d_I^2}{2}\upsilon_1^2+m^2_{Q^I} & -\frac{1}{\sqrt2}(d_I\mu\upsilon_2-d_{S_I}\mu\upsilon_1)\\
-\frac{1}{\sqrt2}(d_I\mu\upsilon_2-d_{S_I}\mu\upsilon_1) & -\frac{1}{12}g'^2(\upsilon^2-2\upsilon_2^2)+\frac{d_I^2}{2}\upsilon_1^2+m^2_{D^I}
\end{pmatrix}.
$$

The fields $\tilde Q_2^I$ and $\tilde D^I$ relate to the two physical (mass) eigenstates $\tilde D_I^i$ $(i=(1,2)$:

$$
\begin{aligned}
\tilde D_I^i&=Z^{i,1}_{D^I}\tilde Q_2^I+Z^{i,2}_{D^I}\tilde D^I,\\
Z^\dagger_{D^I}\mathcal{M}^2_{D^I}Z_{D^I}&=\mathrm{diag}(M^2_{D_I^1},M^2_{D_I^2}).
\end{aligned}\tag{B5}
$$

Appendix C: Expressions of the couplings in $\mathcal{L}_{SSS}$ and $\mathcal{L}_{SSSS}$

In this appendix we give precise expressions of the couplings that appear in $\mathcal{L}_{SSS}$ and $\mathcal{L}_{SSSS}$. The method has been described clearly in text. The results are

$$
A^{kij}_{\mathrm{ec}}=\frac{g^2+g'^2}{4}\Big(\upsilon_1 Z^{k,1}_{\mathrm{even}}Z^{i,1}_{\mathrm{c}}Z^{j,1}_{\mathrm{c}}+\upsilon_2 Z^{k,2}_{\mathrm{even}}Z^{i,2}_{\mathrm{c}}Z^{j,2}_{\mathrm{c}}
$$

$$+\sum_{I=1}^{3}\upsilon_{\tilde{\nu}_I}Z_{\rm even}^{k,2+I}Z_{\rm c}^{i,2+I}Z_{\rm c}^{j,2+I}\Big)$$
$$+\sum_{I=1}^{3}\Big(\frac{g'^2-g^2}{4}+l_I^2\Big)\Big(\upsilon_1Z_{\rm even}^{k,1}Z_{\rm c}^{i,2+I}Z_{\rm c}^{j,2+I}$$
$$+\upsilon_{\tilde{\nu}_I}Z_{\rm even}^{k,2+I}Z_{\rm c}^{i,1}Z_{\rm c}^{j,1}\Big)+\sum_{I=1}^{3}\Big(\frac{g^2}{4}-\frac{1}{2}l_I^2\Big)$$
$$\times\Big\{\upsilon_1Z_{\rm even}^{k,2+I}\Big(Z_{\rm c}^{i,2+I}Z_{\rm c}^{j,2}+Z_{\rm c}^{i,2}Z_{\rm c}^{j,2+I}\Big)$$
$$+\upsilon_{\tilde{\nu}_I}Z_{\rm even}^{k,1}\Big(Z_{\rm c}^{i,2+I}Z_{\rm c}^{j,2}+Z_{\rm c}^{i,2}Z_{\rm c}^{j,2+I}\Big)\Big\}$$
$$+\frac{g^2-g'^2}{4}\Big(\upsilon_1Z_{\rm even}^{k,1}Z_{\rm c}^{i,2}Z_{\rm c}^{j,2}+\upsilon_2Z_{\rm even}^{k,2}Z_{\rm c}^{i,1}Z_{\rm c}^{j,1}$$
$$+\sum_{I=1}^{3}\upsilon_{\tilde{\nu}_I}Z_{\rm even}^{k,2+I}Z_{\rm c}^{i,2}Z_{\rm c}^{j,2}+\upsilon_2Z_{\rm even}^{k,2}Z_{\rm c}^{i,2+I}Z_{\rm c}^{j,2+I}\Big)$$
$$+\sum_{I=1}^{3}\Big[\Big(l_I^2-\frac{g'^2}{2}\Big)\upsilon_{\tilde{\nu}_I}Z_{\rm even}^{k,2+I}Z_{\rm c}^{i,5+I}Z_{\rm c}^{j,5+I}$$
$$+\Big(l_I^2-\frac{g'^2}{2}\Big)\upsilon_1Z_{\rm even}^{k,1}Z_{\rm c}^{i,5+I}Z_{\rm c}^{j,5+I}$$
$$+\frac{g'^2}{2}\upsilon_2Z_{\rm even}^{k,2}Z_{\rm c}^{i,5+I}Z_{\rm c}^{j,5+I}\Big]$$
$$+\frac{g^2}{4}\Big(\upsilon_{\tilde{\nu}_I}Z_{\rm even}^{k,2}+\upsilon_2Z_{\rm even}^{k,2+I}\Big)\Big(Z_{\rm c}^{i,2+I}Z_{\rm c}^{j,2}$$
$$+Z_{\rm c}^{i,2}Z_{\rm c}^{j,2+I}\Big)+\frac{g^2}{4}\Big(\upsilon_1Z_{\rm even}^{k,2}+\upsilon_2Z_{\rm even}^{k,1}\Big)$$
$$\times\Big(Z_{\rm c}^{i,1}Z_{\rm c}^{j,2}+Z_{\rm c}^{i,2}Z_{\rm c}^{j,1}\Big)$$
$$+\frac{1}{\sqrt{2}}l_I\epsilon_3Z_{\rm even}^{k,1}\Big(Z_{\rm c}^{i,4}Z_{\rm c}^{j,2}+Z_{\rm c}^{i,2}Z_{\rm c}^{j,4}\Big)$$
$$+\frac{1}{\sqrt{2}}\sum_{I=1}^{3}l_I\epsilon_IZ_{\rm even}^{k,2}\Big(Z_{\rm c}^{i,5+I}Z_{\rm c}^{j,1}+Z_{\rm c}^{i,1}Z_{\rm c}^{j,5+I}\Big),$$

$$A_{\rm oc}^{kij}=\sum_{I=1}^{3}\Big\{\Big(\frac{g^2}{4}-l_I^2\Big)$$
$$\times\Big[\upsilon_{\tilde{\nu}_I}Z_{\rm odd}^{k,1}\Big(Z_{\rm c}^{i,1}Z_{\rm c}^{j,2+I}-Z_{\rm c}^{i,2+I}Z_{\rm c}^{j,1}\Big)$$
$$+\upsilon_1Z_{\rm odd}^{k,2+I}\Big(Z_{\rm c}^{i,1}Z_{\rm c}^{j,2+I}-Z_{\rm c}^{i,2+I}Z_{\rm c}^{j,1}\Big)\Big]$$
$$+\frac{g^2}{4}\Big(\upsilon_{\tilde{\nu}_I}Z_{\rm odd}^{k,2}+\upsilon_2Z_{\rm odd}^{k,2+I}\Big)$$
$$\times\Big(Z_{\rm c}^{i,2+I}Z_{\rm c}^{j,2}-Z_{\rm c}^{i,2}Z_{\rm c}^{j,2+I}\Big)+\frac{g^2}{4}\Big(\upsilon_{\tilde{\nu}_I}Z_{\rm odd}^{k,2}$$
$$+\upsilon_2Z_{\rm odd}^{k,2+I}\Big)\Big(Z_{\rm c}^{i,2+I}Z_{\rm c}^{j,2}-Z_{\rm c}^{i,2}Z_{\rm c}^{j,2+I}\Big)$$
$$\frac{1}{\sqrt{2}}l_I\epsilon_IZ_{\rm odd}^{k,1}\Big(-Z_{\rm c}^{i,5+I}Z_{\rm c}^{j,2}+Z_{\rm c}^{i,2}Z_{\rm c}^{j,5+I}\Big)$$
$$-\frac{1}{\sqrt{2}}l_I\epsilon_IZ_{\rm odd}^{k,2}\Big(Z_{\rm c}^{i,5+I}Z_{\rm c}^{j,1}-Z_{\rm c}^{i,1}Z_{\rm c}^{j,5+I}\Big)\Big\},$$

$$\mathcal{A}_{\rm ec}^{klij}=\frac{g^2+g'^2}{8}\Big(Z_{\rm even}^{k,1}Z_{\rm even}^{l,1}Z_{\rm c}^{i,1}Z_{\rm c}^{j,1}$$
$$+Z_{\rm even}^{k,2}Z_{\rm even}^{l,2}Z_{\rm c}^{i,2}Z_{\rm c}^{j,2}$$
$$+\sum_{I=1}^{3}Z_{\rm even}^{k,2+I}Z_{\rm even}^{l,2+I}Z_{\rm c}^{i,2+I}Z_{\rm c}^{j,2+I}\Big)$$
$$+\sum_{I=1}^{3}\Big(\frac{g'^2-g^2}{8}+\frac{1}{2}l_I^2\Big)\Big(Z_{\rm even}^{k,1}Z_{\rm even}^{l,1}Z_{\rm c}^{i,2+I}Z_{\rm c}^{j,2+I}$$
$$+Z_{\rm even}^{k,2+I}Z_{\rm even}^{l,2+I}Z_{\rm c}^{i,1}Z_{\rm c}^{j,1}\Big)$$
$$+\frac{g'^2-g^2}{8}\Big[Z_{\rm even}^{k,1}Z_{\rm even}^{l,1}Z_{\rm c}^{i,2}Z_{\rm c}^{j,2}$$
$$+Z_{\rm even}^{k,2}Z_{\rm even}^{l,2}Z_{\rm c}^{i,1}Z_{\rm c}^{j,1}$$
$$+\sum_{I=1}^{3}\Big(Z_{\rm even}^{k,2+I}Z_{\rm even}^{l,2+I}Z_{\rm c}^{i,2}Z_{\rm c}^{j,2}$$
$$+Z_{\rm even}^{k,2}Z_{\rm even}^{l,2}Z_{\rm c}^{i,2+I}Z_{\rm c}^{j,2+I}\Big)\Big]$$
$$+\frac{g^2}{4}\Big[Z_{\rm even}^{k,1}Z_{\rm even}^{l,2}\Big(Z_{\rm c}^{i,2}Z_{\rm c}^{j,1}+Z_{\rm c}^{i,1}Z_{\rm c}^{j,2}\Big)$$
$$+\sum_{I=1}^{3}Z_{\rm even}^{k,1}Z_{\rm even}^{l,2+I}\Big(Z_{\rm c}^{i,2+I}Z_{\rm c}^{j,1}+Z_{\rm c}^{i,1}Z_{\rm c}^{j,2+I}\Big)$$
$$+\sum_{I=1}^{3}Z_{\rm even}^{k,2}Z_{\rm even}^{l,2+I}\Big(Z_{\rm c}^{i,2+I}Z_{\rm c}^{j,2}+Z_{\rm c}^{i,2}Z_{\rm c}^{j,2+I}\Big)\Big]$$
$$-\sum_{I=1}^{3}\Big[\frac{l_I^2}{2}Z_{\rm even}^{k,1}Z_{\rm even}^{l,2+I}\Big(Z_{\rm c}^{i,2+I}Z_{\rm c}^{j,1}+Z_{\rm c}^{i,1}Z_{\rm c}^{j,2+I}\Big)$$
$$-\frac{g'^2}{4}\Big(Z_{\rm even}^{k,1}Z_{\rm even}^{l,1}Z_{\rm c}^{i,5+I}Z_{\rm c}^{j,5+I}$$
$$-Z_{\rm even}^{k,2}Z_{\rm even}^{l,2}Z_{\rm c}^{i,5+I}Z_{\rm c}^{j,5+I}$$
$$+Z_{\rm even}^{k,2+I}Z_{\rm even}^{l,2+I}Z_{\rm c}^{i,5+I}Z_{\rm c}^{j,5+I}\Big)$$
$$+\frac{l_I^2}{2}Z_{\rm even}^{k,1}Z_{\rm even}^{l,1}Z_{\rm c}^{i,5+I}Z_{\rm c}^{j,5+I}\Big],$$

$$\mathcal{A}_{\rm oc}^{klij}=\frac{g^2+g'^2}{8}\Big(Z_{\rm odd}^{k,1}Z_{\rm odd}^{l,1}Z_{\rm c}^{i,1}Z_{\rm c}^{j,1}+Z_{\rm odd}^{k,2}Z_{\rm odd}^{l,2}Z_{\rm c}^{i,2}Z_{\rm c}^{j,2}$$
$$+\sum_{I=1}^{3}Z_{\rm odd}^{k,2+I}Z_{\rm odd}^{l,2+I}Z_{\rm c}^{i,2+I}Z_{\rm c}^{j,2+I}\Big)$$
$$+\sum_{I=1}^{3}\Big(\frac{g'^2-g^2}{8}+\frac{1}{2}l_I^2\Big)\Big(Z_{\rm odd}^{k,1}Z_{\rm odd}^{l,1}Z_{\rm c}^{i,2+I}Z_{\rm c}^{j,2+I}$$
$$+Z_{\rm odd}^{k,2+I}Z_{\rm odd}^{l,2+I}Z_{\rm c}^{i,1}Z_{\rm c}^{j,1}\Big)$$
$$+\frac{g'^2-g^2}{8}\Big\{Z_{\rm odd}^{k,1}Z_{\rm odd}^{l,1}Z_{\rm c}^{i,2}Z_{\rm c}^{j,2}$$

$$+Z_{\rm odd}^{k,2}Z_{\rm odd}^{l,2}Z_{\rm c}^{i,1}Z_{\rm c}^{j,1}$$
$$+\sum_{I=1}^{3}\Big(Z_{\rm odd}^{k,2+I}Z_{\rm odd}^{l,2+I}Z_{\rm c}^{i,2}Z_{\rm c}^{j,2}$$
$$+Z_{\rm odd}^{k,2}Z_{\rm odd}^{l,2}Z_{\rm c}^{i,2+I}Z_{\rm c}^{j,2+I}\Big)\Big\}$$
$$+\frac{g^2}{4}\Big\{Z_{\rm odd}^{k,1}Z_{\rm odd}^{l,2}\Big(Z_{\rm c}^{i,2}Z_{\rm c}^{j,1}+Z_{\rm c}^{i,1}Z_{\rm c}^{j,2}\Big)$$
$$+\sum_{I=1}^{3}\Big[Z_{\rm odd}^{k,1}Z_{\rm odd}^{l,3}\Big(Z_{\rm c}^{i,3}Z_{\rm c}^{j,1}+Z_{\rm c}^{i,1}Z_{\rm c}^{j,3}\Big)$$
$$+Z_{\rm odd}^{k,2}Z_{\rm odd}^{l,3}\Big(Z_{\rm c}^{i,3}Z_{\rm c}^{j,2}+Z_{\rm c}^{i,2}Z_{\rm c}^{j,3}\Big)\Big]\Big\}$$
$$-\sum_{I=1}^{3}\Big\{\frac{l_I^2}{2}Z_{\rm odd}^{k,1}Z_{\rm odd}^{l,2+I}\Big(Z_{\rm c}^{i,2+I}Z_{\rm c}^{j,1}+Z_{\rm c}^{i,1}Z_{\rm c}^{j,2+I}\Big)$$
$$-\frac{g'^2}{4}\Big(Z_{\rm odd}^{k,1}Z_{\rm odd}^{l,1}Z_{\rm c}^{i,5+I}Z_{\rm c}^{j,5+I}$$
$$-Z_{\rm odd}^{k,2}Z_{\rm odd}^{l,2}Z_{\rm c}^{i,5+I}Z_{\rm c}^{j,5+I}$$
$$+Z_{\rm odd}^{k,2+I}Z_{\rm odd}^{l,2+I}Z_{\rm c}^{i,5+I}Z_{\rm c}^{j,5+I}\Big)$$
$$+\frac{l_I^2}{2}Z_{\rm odd}^{k,1}Z_{\rm odd}^{l,1}Z_{\rm c}^{i,5+I}Z_{\rm c}^{j,5+I}\Big\},$$

$$\mathcal{A}_{\rm eoc}^{klij}=\sum_{I=1}^{3}\Big[\Big(\frac{g^2}{8}+\frac{1}{2}l_I^2\Big)\Big(Z_{\rm even}^{k,2+I}Z_{\rm odd}^{l,1}+Z_{\rm even}^{k,1}Z_{\rm odd}^{l,2+I}\Big)$$
$$\times\Big(Z_{\rm c}^{i,1}Z_{\rm c}^{j,2+I}-Z_{\rm c}^{i,2+I}Z_{\rm c}^{j,1}\Big)$$
$$-\frac{g^2}{8}\Big(Z_{\rm even}^{k,2}Z_{\rm odd}^{l,2+I}+Z_{\rm even}^{k,2+I}Z_{\rm odd}^{l,2}\Big)$$
$$\times\Big(Z_{\rm c}^{i,2+I}Z_{\rm c}^{j,2}-Z_{\rm c}^{i,2}Z_{\rm c}^{j,2+I}\Big)\Big]$$
$$-\frac{g^2}{8}\Big(Z_{\rm even}^{k,1}Z_{\rm odd}^{l,2}+Z_{\rm even}^{k,2}Z_{\rm odd}^{l,1}\Big)$$
$$\times\Big(Z_{\rm c}^{i,1}Z_{\rm c}^{j,2}-Z_{\rm c}^{i,2}Z_{\rm c}^{j,2}\Big),$$

$$\mathcal{A}_{\rm cc}^{ijkl}=\frac{g^2+g'^2}{8}\Big[\sum_{m,n=1}^{5}Z_{\rm c}^{i,m}Z_{\rm c}^{j,m}Z_{\rm c}^{k,n}Z_{\rm c}^{l,n}$$
$$+2\sum_{I=1}^{3}Z_{\rm c}^{i,2+I}Z_{\rm c}^{j,2+I}\Big(Z_{\rm c}^{k,1}Z_{\rm c}^{l,1}-Z_{\rm c}^{k,2}Z_{\rm c}^{l,2}\Big)$$
$$-2Z_{\rm c}^{i,1}Z_{\rm c}^{j,1}Z_{\rm c}^{k,2}Z_{\rm c}^{l,2}\Big]+\frac{g'^2}{2}\Big[\sum_{I=1}^{3}Z_{\rm c}^{i,5+I}Z_{\rm c}^{j,5+I}$$
$$\times\Big(-Z_{\rm c}^{k,5+I}Z_{\rm c}^{l,5+I}-Z_{\rm c}^{k,2+I}Z_{\rm c}^{l,2+I}\Big)$$
$$-\sum_{I=1}^{3}Z_{\rm c}^{i,5+I}Z_{\rm c}^{j,5+I}Z_{\rm c}^{k,1}Z_{\rm c}^{l,1}\Big]$$
$$+\sum_{I=1}^{3}l_I^2Z_{\rm c}^{i,5+I}Z_{\rm c}^{j,5+I}Z_{\rm c}^{k,1}Z_{\rm c}^{l,1},$$

where the mixing matrices $Z_{\rm even}$, $Z_{\rm odd}$ and $Z_{\rm c}$ are defined as in (18), (22) and (30).

References

1. For reviews see, for example, H.P. Nilles, Phys. Rep. **110**, 1 (1984); J. Wess, J. Bagger, Supersymmetry and Supergravity (Princeton University Press 1992)
2. J. Erler, J. Feng, N. Polonsky, Phys. Rev. Lett. **78**, 3012 (1997); L. Hall, M. Suzuki, Nucl. Phys. B **231**, 419 (1984); I.H. Lee, Nucl. Phys. B **246**, 120 (1984); J. Ellis, G. Gelmini, C. Jarlskog, G.G. Ross, J.W.F. Valle, Phys. Lett. B **151**, 375 (1985); S. Dawson, Nucl. Phys. B **261**, 297 (1985); R. Barbieri, A. Masiero, Nucl. Phys. B **267**, 679 (1986); S. Dimopoulos, L.J. Hall, Phys. Lett. B **207**, 210 (1987)
3. S. Roy, B. Mukhopadhyaya, Phys. Rev. D **55**, 7020 (1997); H.P. Nilles, Polonsky, Nucl. Phys. B **484**, 33 (1997); R. Hempfling, Nucl. Phys. B **478** 3 (1996)
4. G. Bhattacharyya, D. Choudhury, K. Sridhar, Phys. Lett. B **349**, 118 (1995); **355**, 193 (1995); G. Bhattacharyya, J. Ellis, K. Sridhar, Mod. Phys. Lett. A **10**, 1583 (1995); A.Yu. Smirnov, F. Vissani, Phys. Lett. B **380**, 317 (1996); K. Agashe, M. Graesser, Phys. Rev. D **54**, 4445 (1996)
5. R. Barbier, C. Berat et al., hep-ph/9810232
6. D. Choudhury, P. Roy, Phys. Lett. B **378**, 153 (1996); J.-H. Jang, J.K. Kim, J.S. Lee, Phys. Rev. D **55**, 7296 (1997); J.K. Kim, P. Ko, D.-G. Lee, Phys. Rev. D **56**, 100 (1997); V. Barger, G.F. Giudice, T. Han, Phys. Rev. D **40**, 2987 (1989); J. Ellis, G. Bhattacharyya, K. Sridhar, Mod. Phys. Lett. A **10**, 1583 (1995)
7. S. Davidson, J. Ellis, Phys. Lett. B **390**, 210 (1997); Phys. Rev. D **56**, 4182 (1997); M. Carena, S. Pokorski, C.E.M. Wagner, hep-ph/9801251
8. V. Barger et al., Phys. Rev. D **53**, 6407 (1996)
9. B. de Carlos, P.L. White, Phys. Rev. D **54**, 3427 (1996)
10. L. Hall, M. Suzuki, Nucl. Phys. B **231**, 419 (1984); Y. Grossman, Z. Ligeti, E. Nardi, Nucl. Phys. B **465**, 369 (1996); C.H. Chen, C.Q. Geng, C.C. Lin, Phys. Rev. D **56**, 6856 (1997); M. Hirsch, H.V. Klapdor-Kleingrothaus, S.G. Kovalenko, Phys. Rev. Lett. **75**, 17 (1995); ibid., Phys. Rev. D **53**, 1379 (1996)
11. C.-H. Chang, T.-F. Feng, L.-Y. Shang, hep-ph/9806505
12. Y. Grossman, H.E. Haber, hep-ph/9810536
13. H.E. Haber, G.L. Kane, Phys. Rep. **117**, 75 (1985); J. Rosiek, Phys. Rev. D **41**, 3464 (1990)
14. Particle Data Group, Eur. Phys. J. C **3**, 1 (1998)
15. M. Nowakowski, A. Pilaftsis, Nucl. Phys. B **461**, 19 (1996); A. Joshipra, M. Nowakowski, Phys. Rev. D **51**, 2421 (1997)
16. C. Itzykson, J.-B. Zuber, Quantum Field Theory (McGraw-Hill, New York 1980)
17. J.A. Grifols, A. Mendez, Phys Rev. D **22**, 1725 (1977)
18. T. Gajdosik, Ph.D. dissertation (Technischen Universität 1995)
19. J. Gunion, H.E. Haber, Nucl. Phys. B **272**, 1 (1986)
20. D. Guetta, Phys. Rev. D **58**, 116008 (1998); D. Choudhury, B. Dutta, A. Kundu, hep-ph/9812209; T.-F. Feng, hep-ph/9808379
21. M.A. Diaz, J.C. Romao, J.W.F. Valle, Nucl. Phys. B **524**, 23 (1998)

22. S.Y. Choi, E.J. Chun, S.K. Kang, J.S. Lee, hep-ph/9903465; A. Datta, B. Mukhopadhyaya, S. Roy, hep-ph/9905549
23. M. Hirsch, J.W.F. Valle, hep-ph/9812463; A. Faessler, S. Kovalenko, hep-ph/9712535; A. Faessler, F. Simkovic, hep-ph/9901215
24. E. Ma, M. Raidal, U. Servant, hep-ph/9902220; F. de Campos, M.A. Diaz, O.J.P. Eboli, M.B. Magro, L. Navarro, W. Porod, D.A. Restrepo, J.W.F. Valle, hep-ph/9903245; B. Mukhopadhyaya, S. Roy, hep-ph/9903418; L. Navarro, W. Porod, J.W.F. Valle, hep-ph/9903474; G. Bhattachayya, A. Datta, hep-ph/9903490; E.L. Berger, B.W. Harris, Z. Sullivan, hep-ph/9903549; M.A. Diaz, hep-ph/9905422; F. Borzumati, J.L. Kneur, N. Polonsky, hep-ph/9905443
25. J.M. Yang, B.-L. Young, X. Zhang, Phys. Rev. D **58**, 055001 (1998); A. Datta, J.M. Yang, B.-L. Young, X. Zhang, Phys. Rev. D **56**, 3107 (1997)
26. H.E. Haber, R. Hempfling, Phys. Rev. D **48**, 4280 (1993); J.A. Casas, J.R. Espinosa, H.E. Haber, Nucl. Phys B **526**, 3 (1998)
27. S. Heinemeyer, W. Hollik, G. Weiglein, hep-ph/9903404; S. Heinemeyer, W. Hollik, G. Weiglein, hep-ph/9903504; S. Heinemeyer, W. Hollik, G. Weiglein, hep-ph/9812472
28. E. Nardi, Phys. Rev. D **55**, 5772 (1997)
29. M. Bisset, O.C.W. Kong, C. Macesanu, L.H. Orr, Phys. Lett. B **430**, 274 (1998)